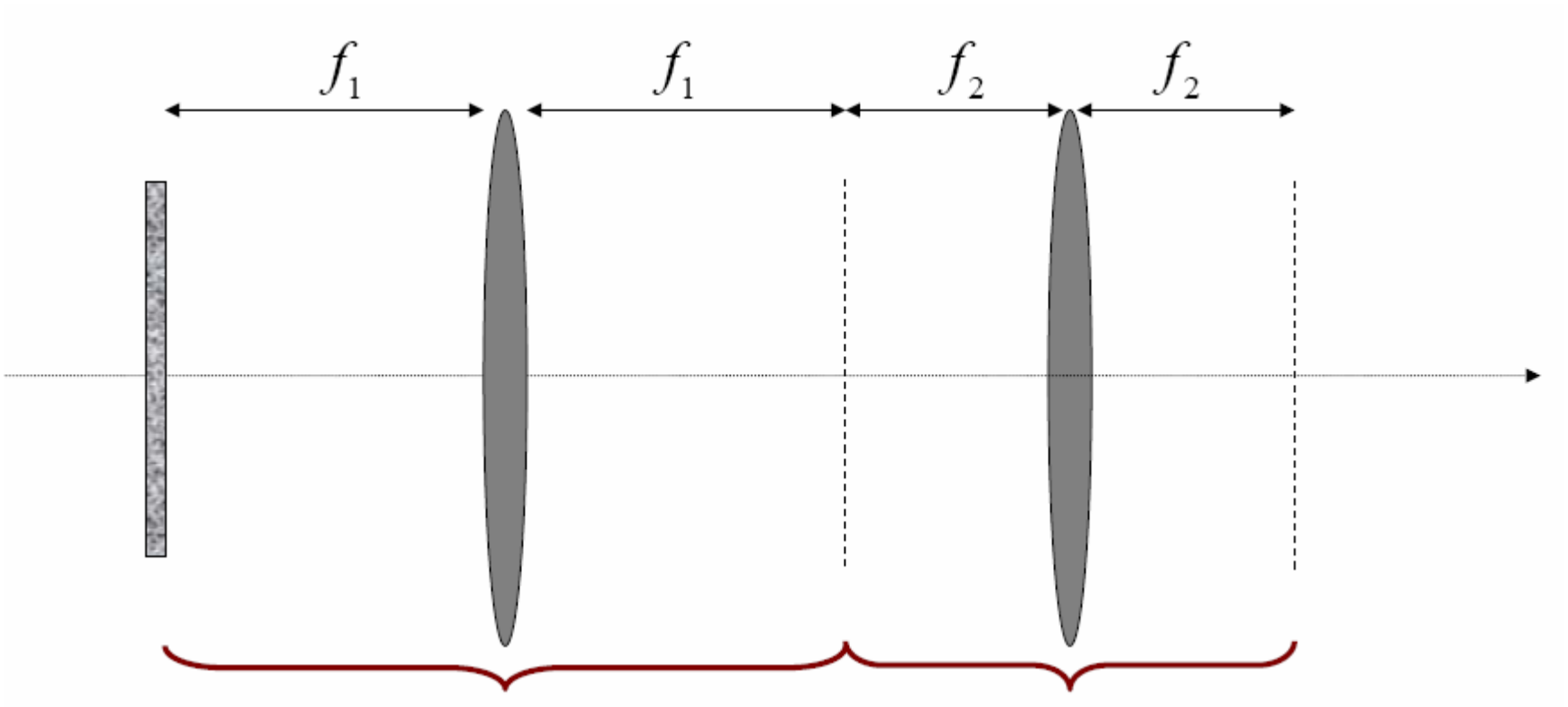
今日内容

相干光成像

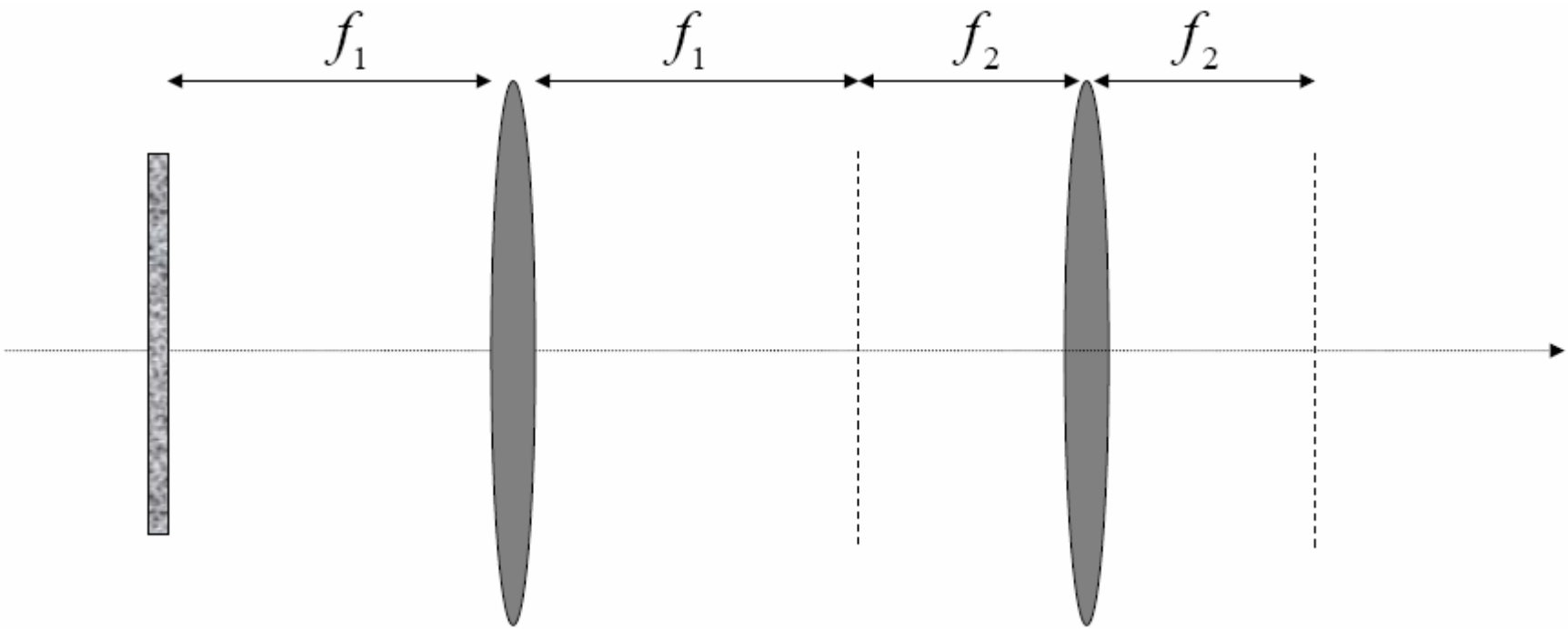
- 相干像的形成
  - 空域描述：脉冲响应
  - 空间频域描述：相干传递函数

VISIT…

4F系统



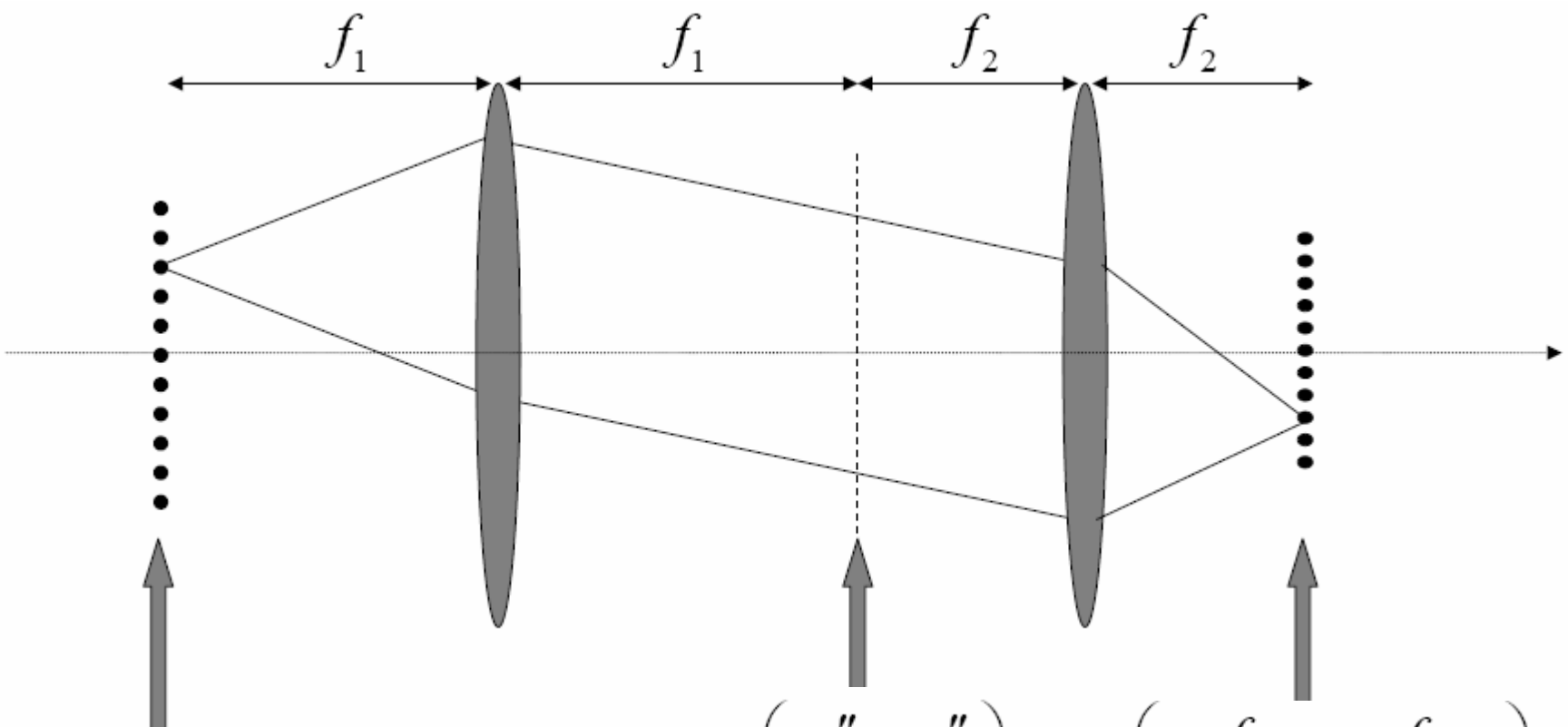
4F系统



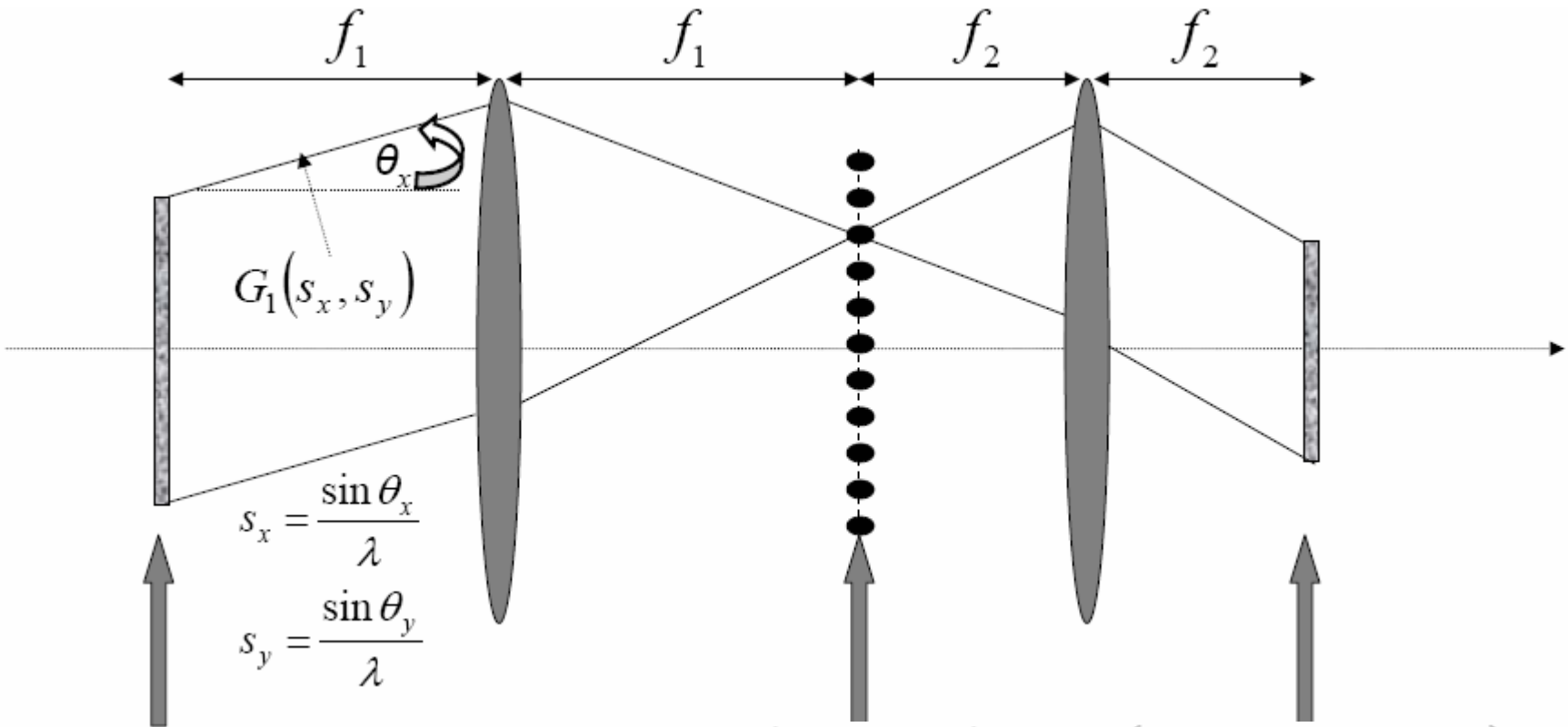
定理：

$$\Im\{\Im\{g(x,y)\}\} = g(-x,-y)$$

4F系统


f_1
f_1
f_2
f_2
$g_1(x,y)$
$G_1\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)$
$g_1\left(-\frac{f_1}{f_2}x',-\frac{f_1}{f_2}y'\right)$
物平面
傅里叶平面
像平面

4F系统

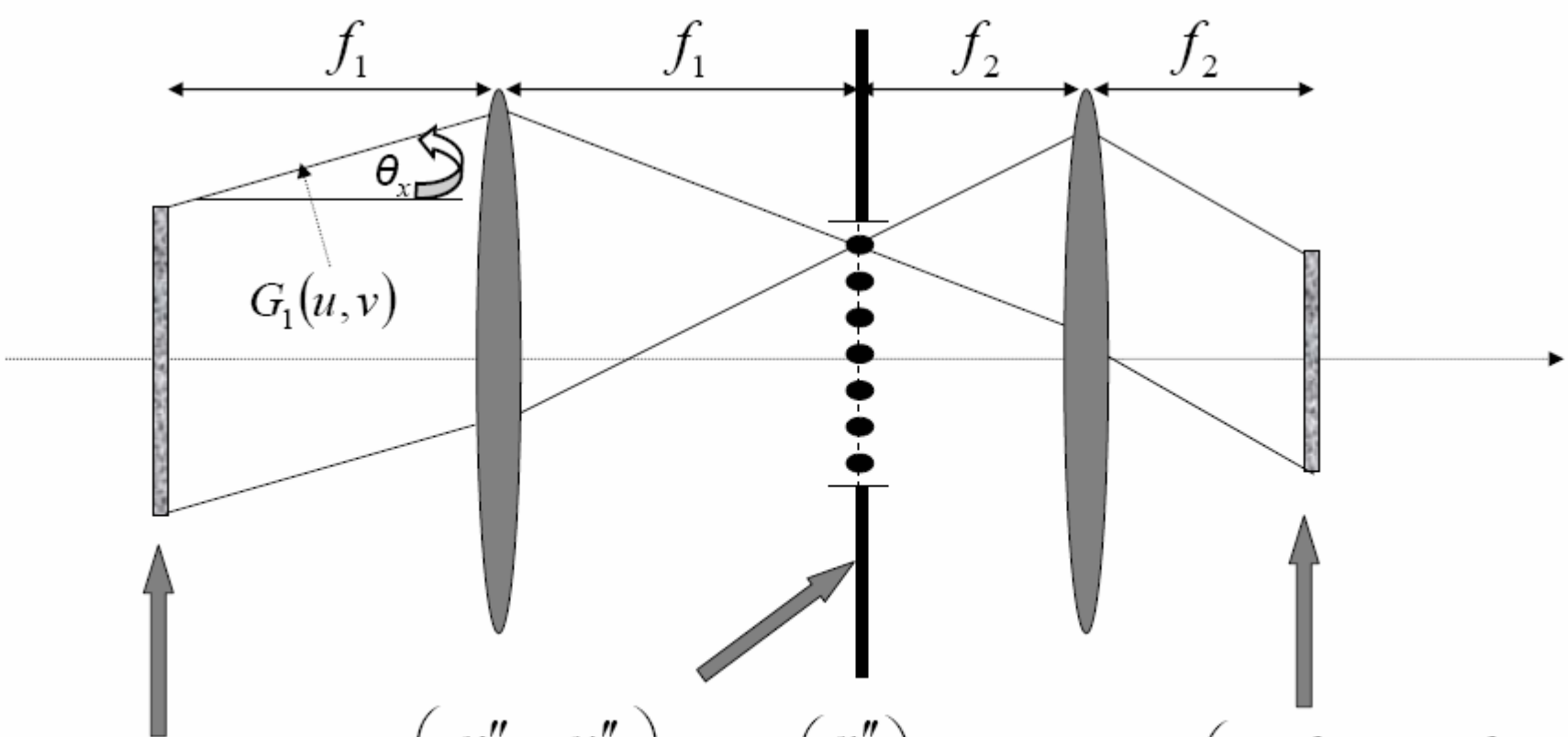


物平面

傅里叶平面

像平面

带有FP孔径的4F系统



$g_1(x,y)$

物平面

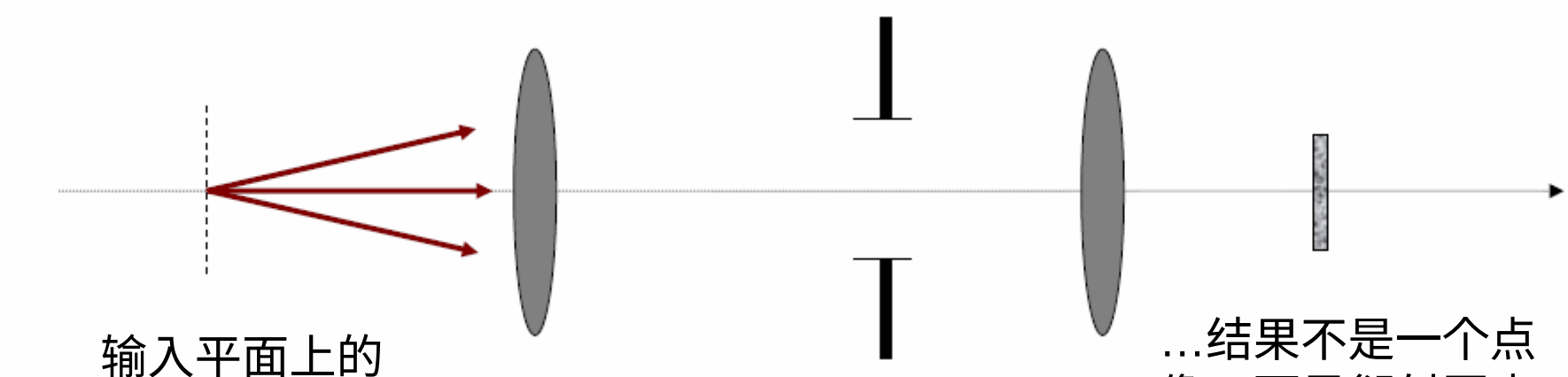
$$G_1\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)\times \mathrm{circ}\left(\frac{r''}{R}\right)$$

傅里叶平面：孔径受限

$$(g_1 * h)\left(-\frac{f_1}{f_2}x',-\frac{f_1}{f_2}y'\right)$$

像平面：被模糊化
（即低通滤波）

脉冲响应和传递函数



输入平面上的
一个点源

…结果不是一个点
像，而是衍射图案
$h(x',y')$

物面处的点源 $\leftrightarrow$ δ函数 $\delta(x,y)$

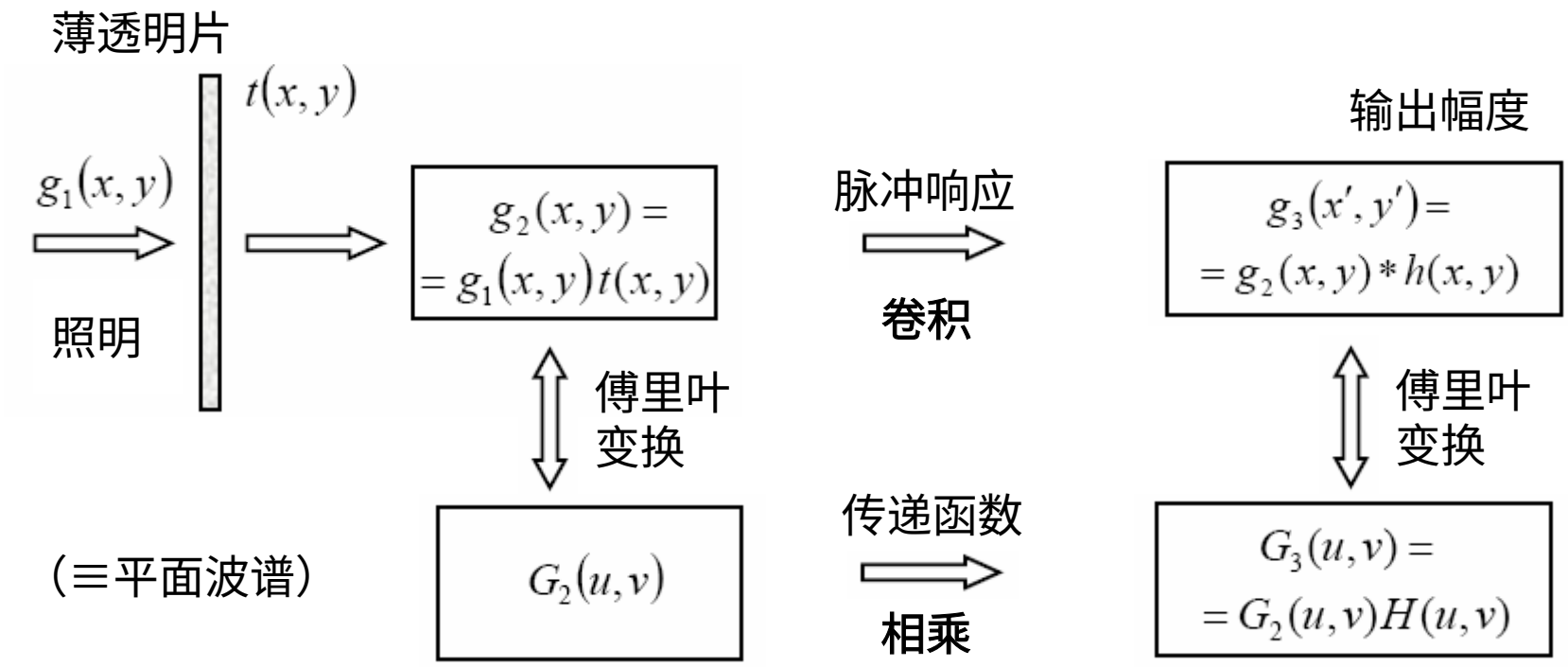
$h(x',y')$ 是系统的脉冲响应函数

更一般地，

$h(x',y')$ **被称为相干点扩散函数（相干PSF）**

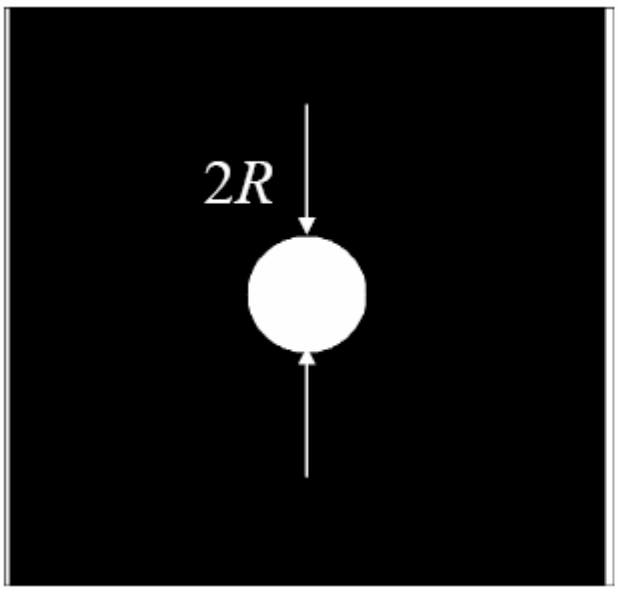
相干成像

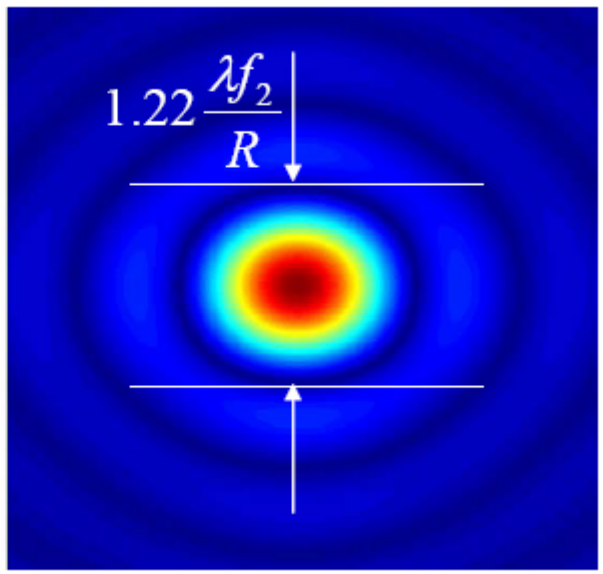
线性平移不变系统



传递函数 $H(u,v)$ ：即光瞳函数

圆孔的传递函数和脉冲响应





传递函数：
圆孔

$$\mathrm{circ}\left(\frac{r''}{R}\right)$$

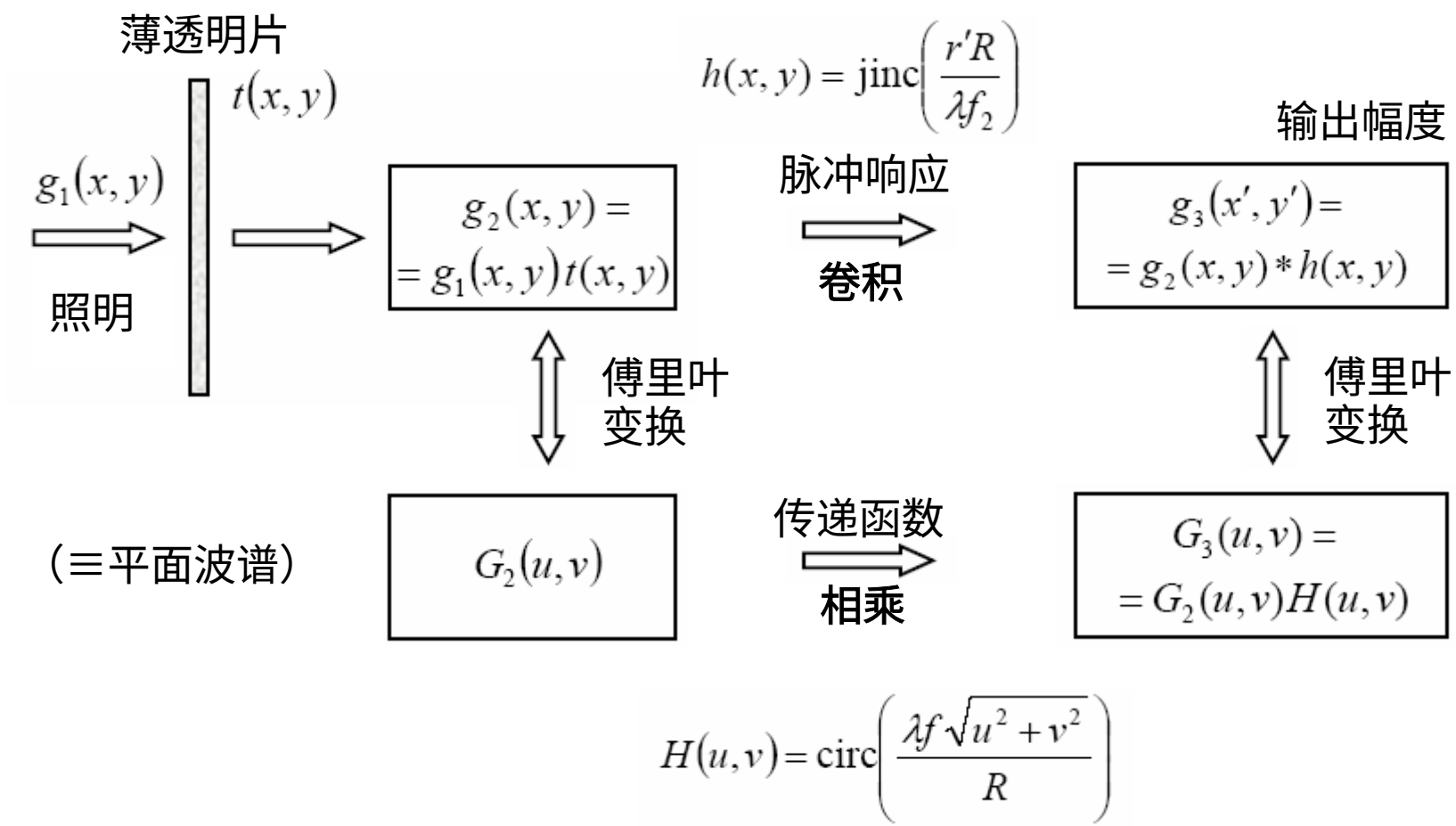
脉冲响应：
爱里函数

$$\mathrm{jinc}\left(\frac{r'R}{\lambda f_2}\right)$$

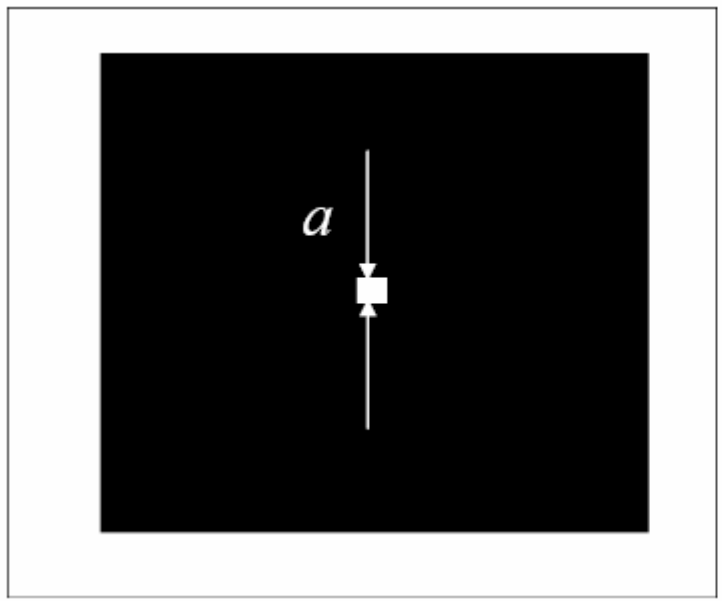
相干成像

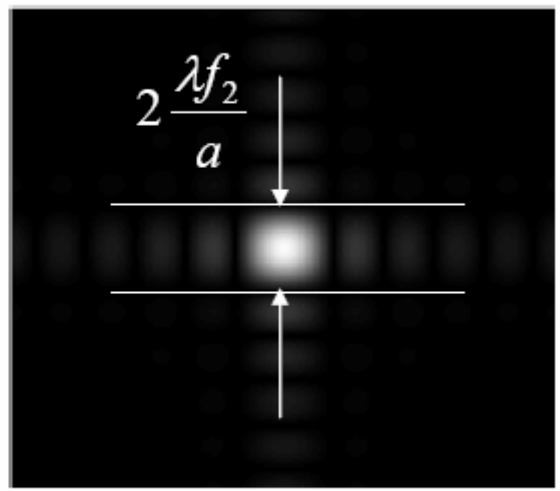
线性平移不变系统

范例：在傅里叶平面上是*圆孔的*4F系统



矩形孔的传递函数和脉冲响应





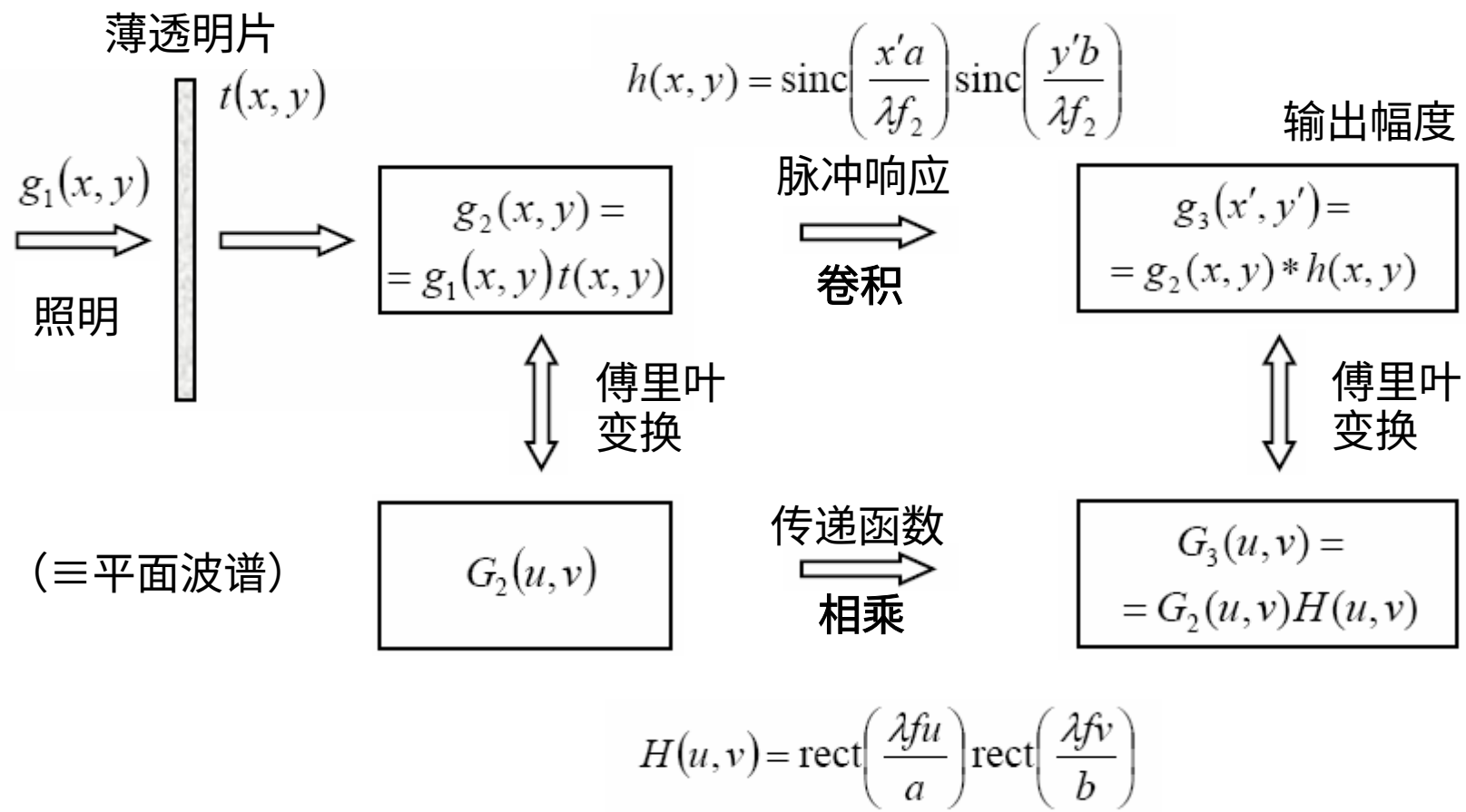
$$\mathrm{rect}\left(\frac{x''}{a}\right)\mathrm{rect}\left(\frac{y''}{b}\right)$$

$$\mathrm{sinc}\left(\frac{x'a}{\lambda f_2}\right)\mathrm{sinc}\left(\frac{y'b}{\lambda f_2}\right)$$

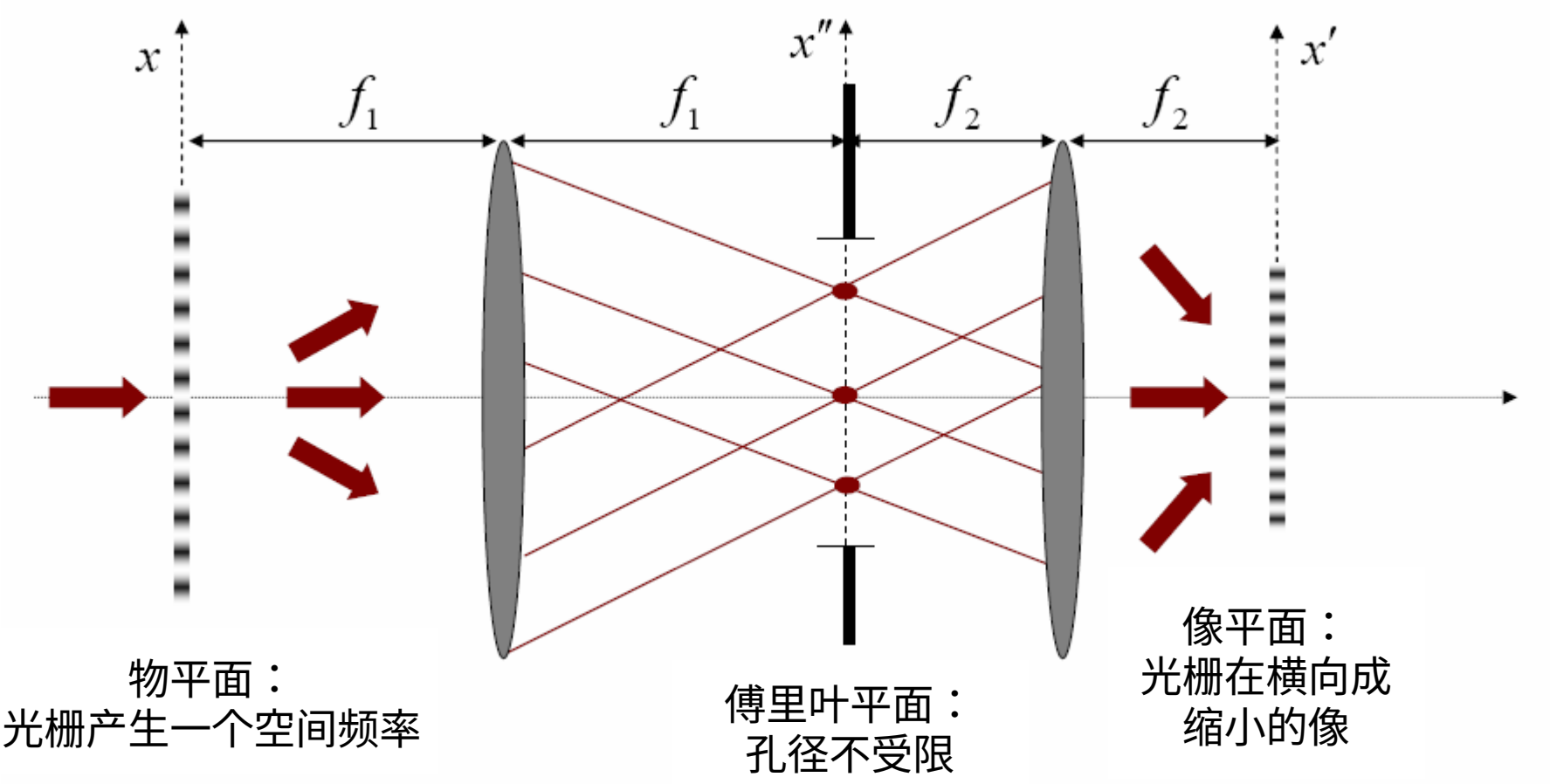
相干成像

线性平移不变系统

例：在傅里叶平面上是*矩形孔的*4F系统

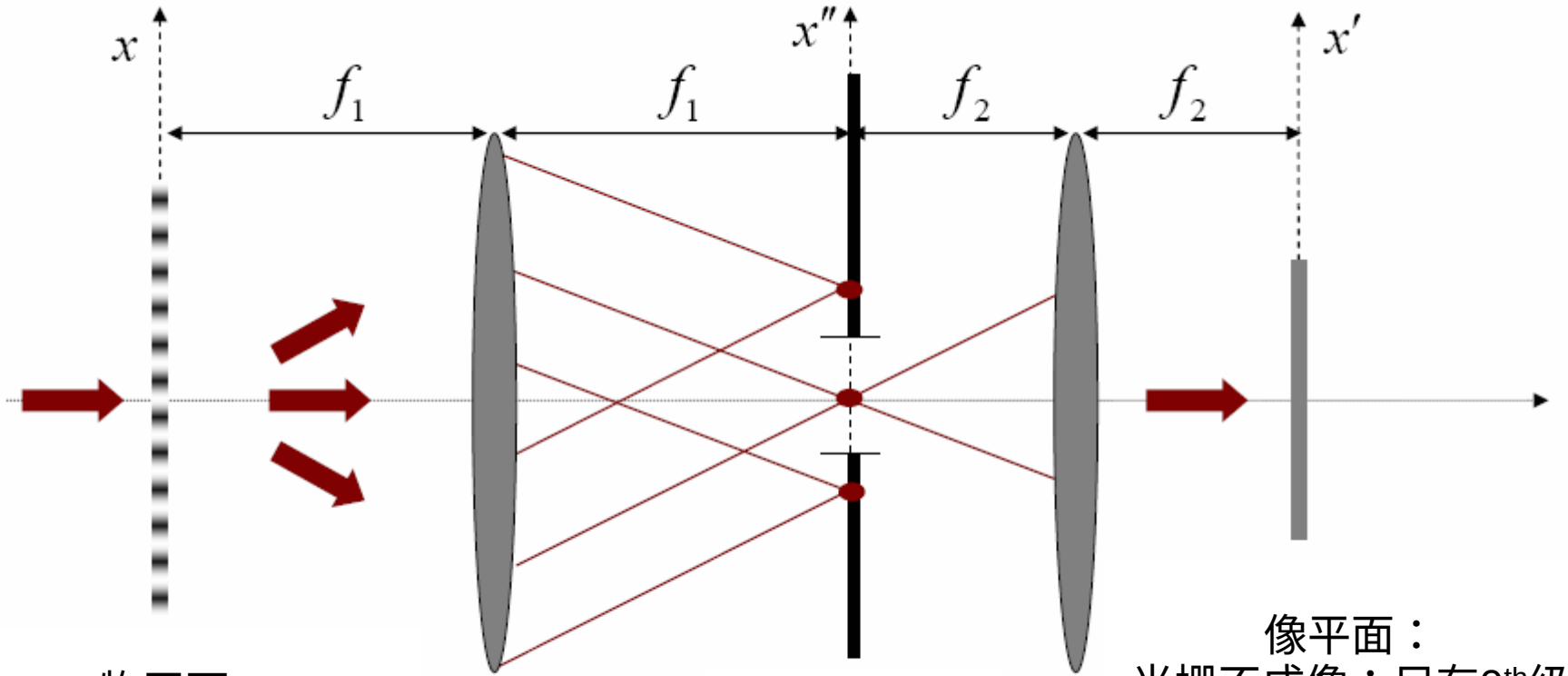


孔径受限的空间滤波



（各级均通过）

孔径受限的空间滤波



物平面：
光栅产生一个空间频率

傅里叶平面：
孔径受限

像平面：
光栅不成像；只有0th级
（直流分量）存在

（某些级被挡掉）

空间频率剪切

输入透明片后的场

$$g_{\text{in}}(x)=\frac{1}{2}\left[1+\cos(2\pi u_0 x)\right]\Rightarrow G_{\text{in}}(u)=\frac{1}{2}\left[\delta(u)+\frac{1}{2}\delta(u-u_0)+\frac{1}{2}\delta(u+u_0)\right]$$

滤波前的场

$$g_{\text{f}-}(x'')=\frac{1}{2}\left[\delta(x'')+\frac{1}{2}\delta(x''-\lambda f_1 u_0)+\frac{1}{2}\delta(x''-\lambda f_1 v_0)\right]$$

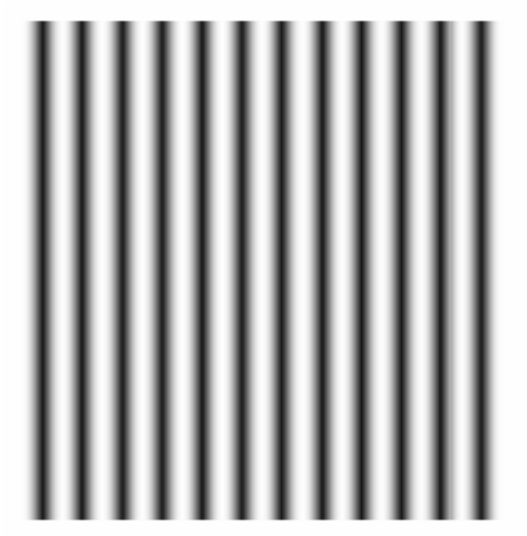
滤波后的场

$$g_{\text{f}+}(x'')=\frac{1}{2}\delta(x'')$$

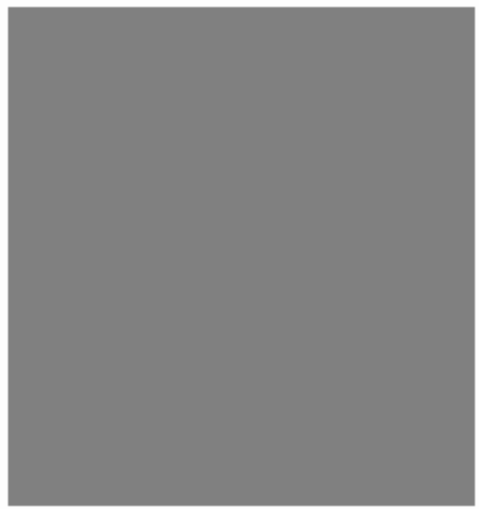
输出场（像平面）

$$G_{\text{out}}(u)=\frac{1}{2}\delta(u)\Rightarrow g_{\text{out}}(x')=\frac{1}{2}$$

空间滤波的效果



圆孔傅里叶
平面滤波器

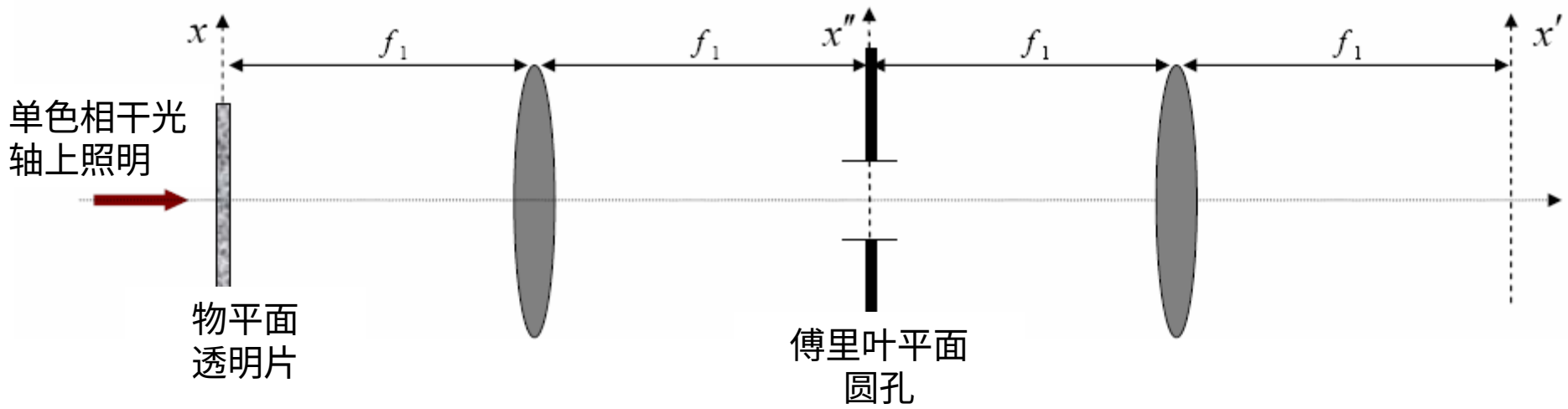


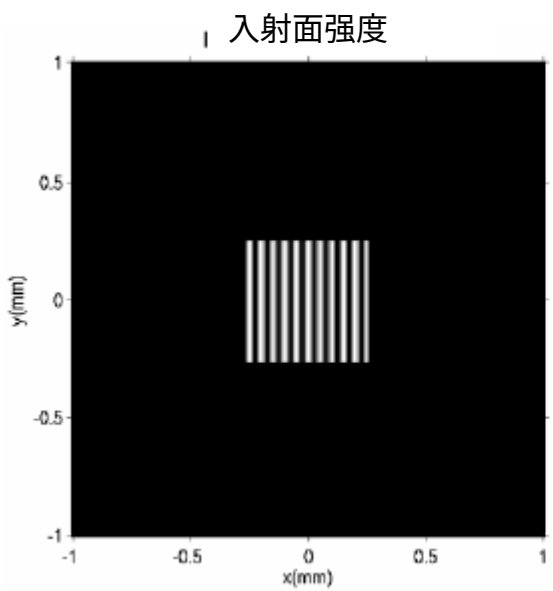
原物（强度是正弦型空间变化，即光栅）

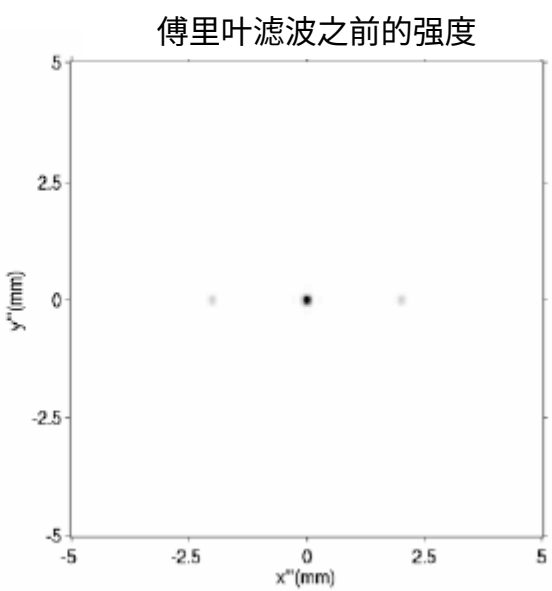
频率滤波的图像（空间变化被模糊，只剩下平均强度）

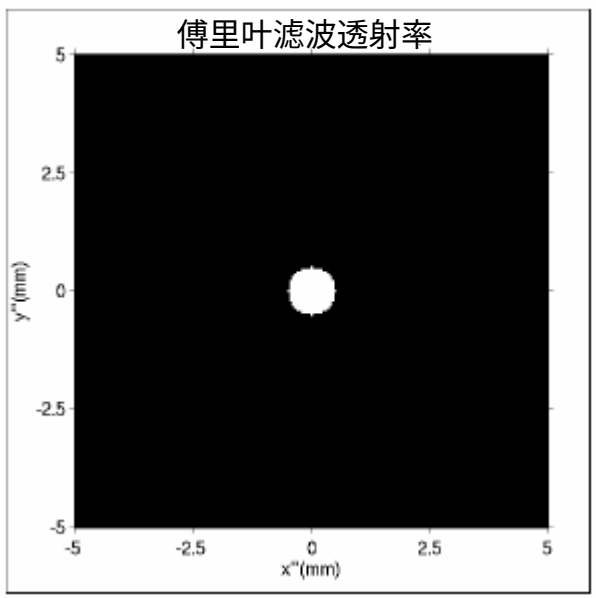
空间频率剪切

f_1=20cm
λ=0.5μm

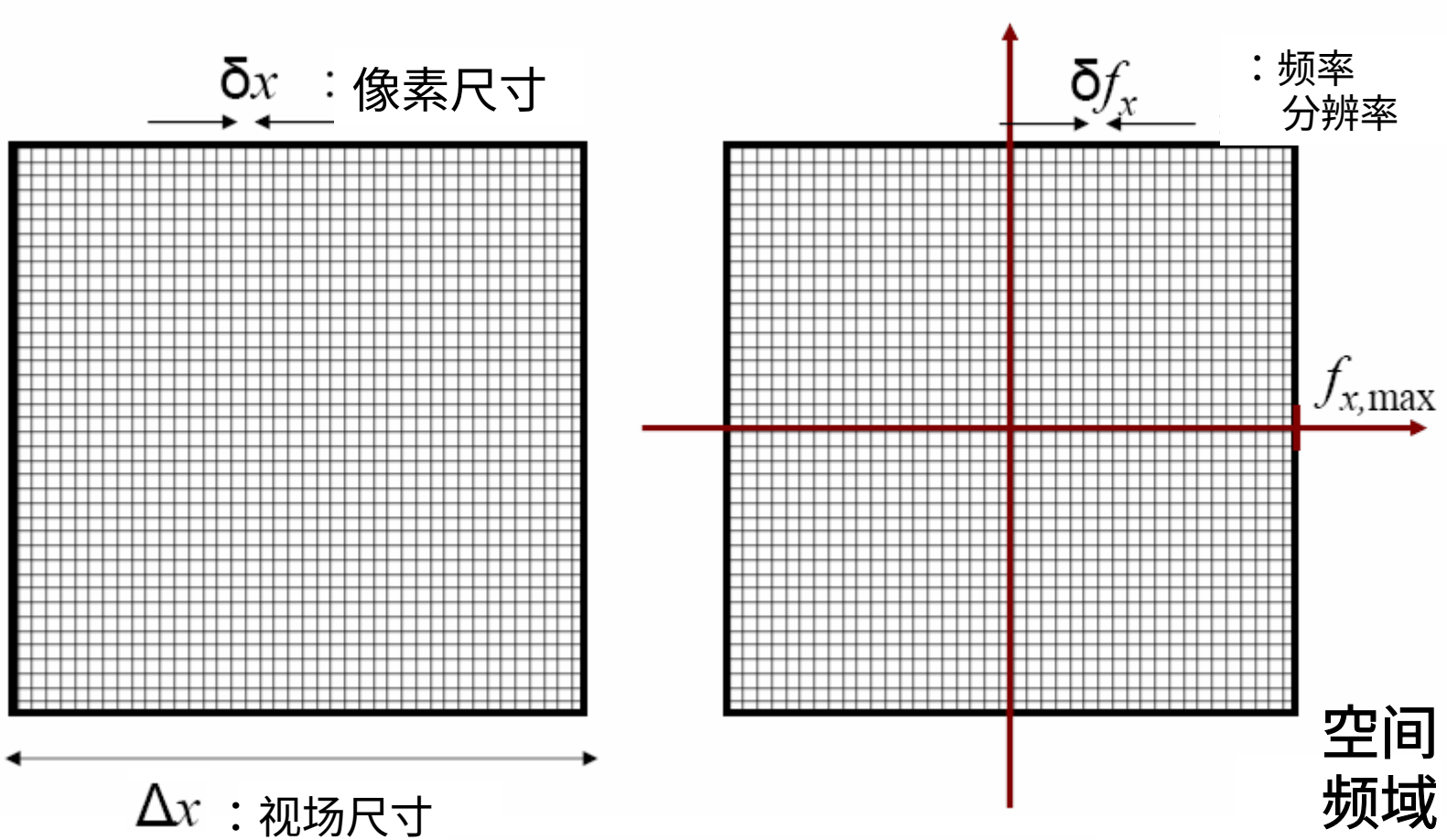








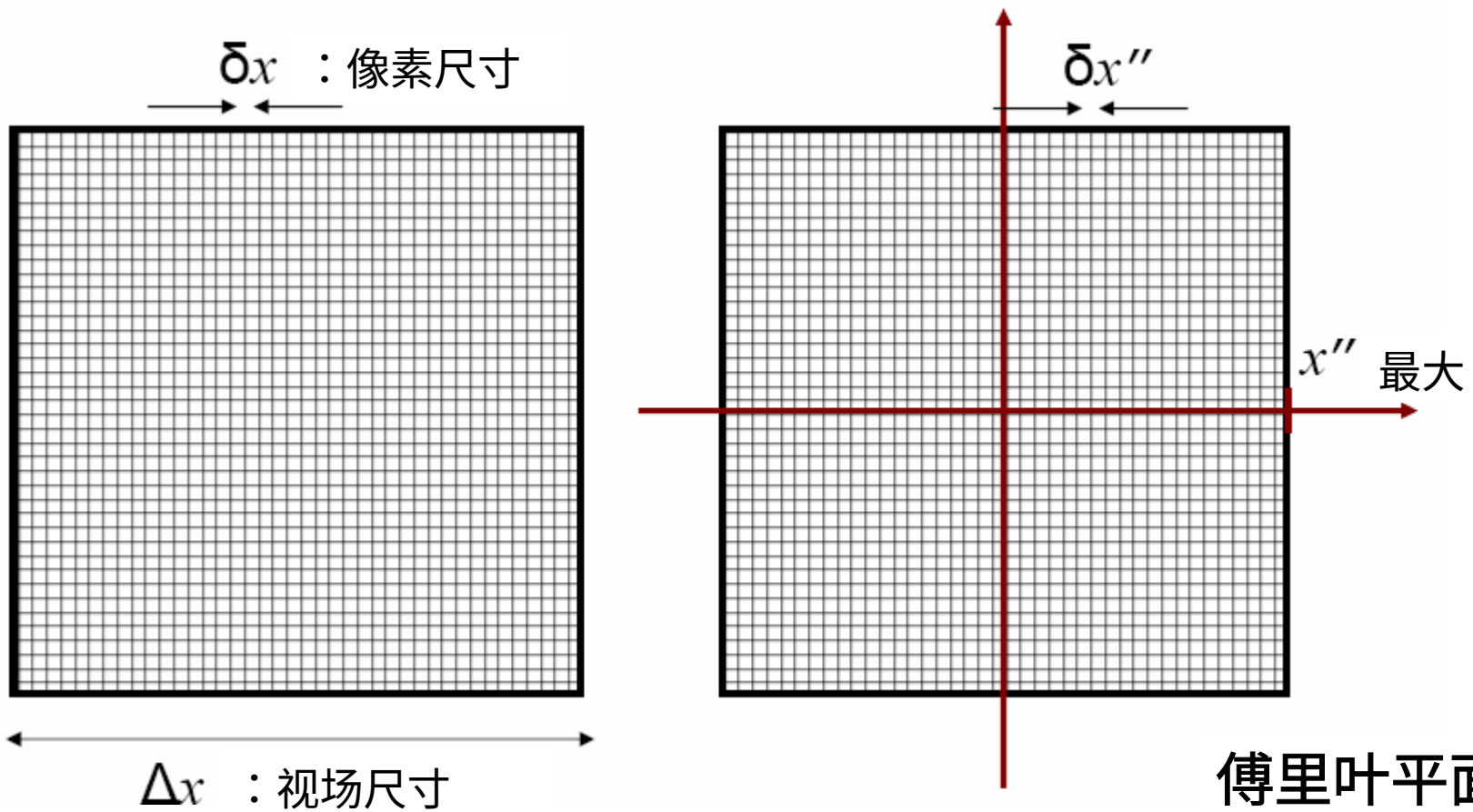
空间傅里叶坐标变换



Nyquist 关系式：

$$f_{x,\max} = \frac{1}{2\delta x} \qquad \delta f_x = \frac{1}{2\Delta x}$$

4F坐标变换

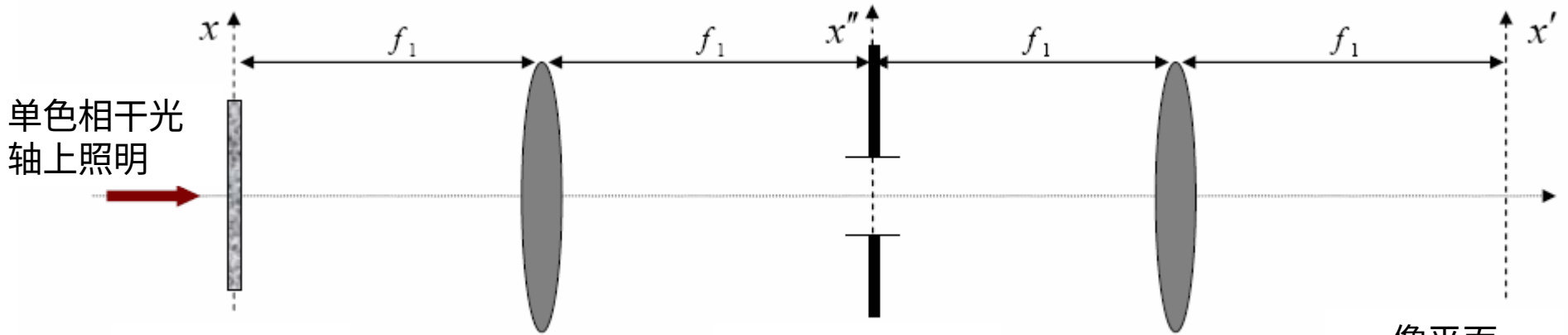


Nyquist 关系式：

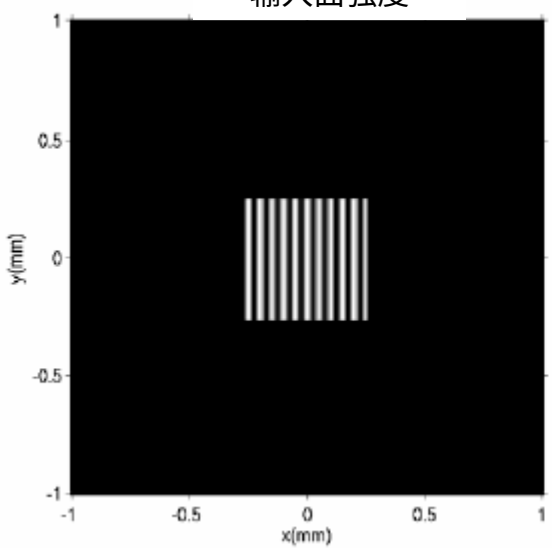
$$x''_{\max} = \frac{\lambda f}{2\delta x} \qquad \delta x'' = \frac{\lambda f}{2\Delta x}$$

空间频率剪切

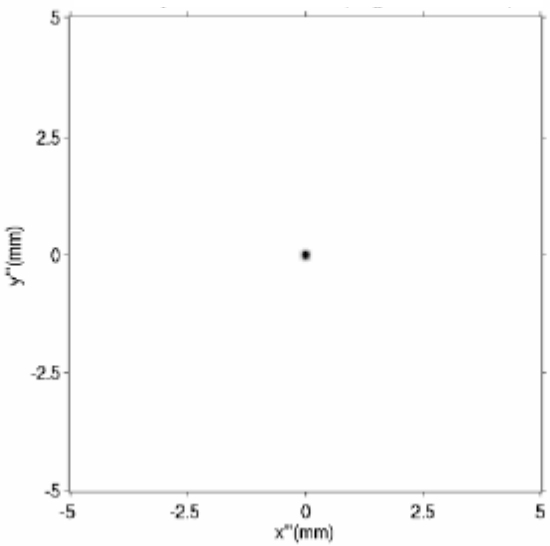
f_1=20cm
λ=0.5μm



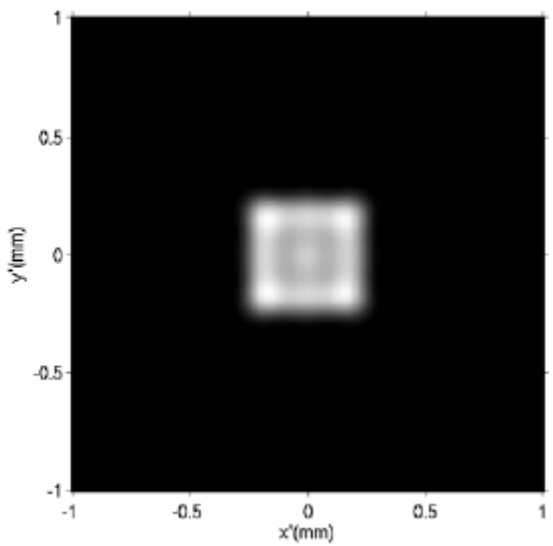
输入面强度



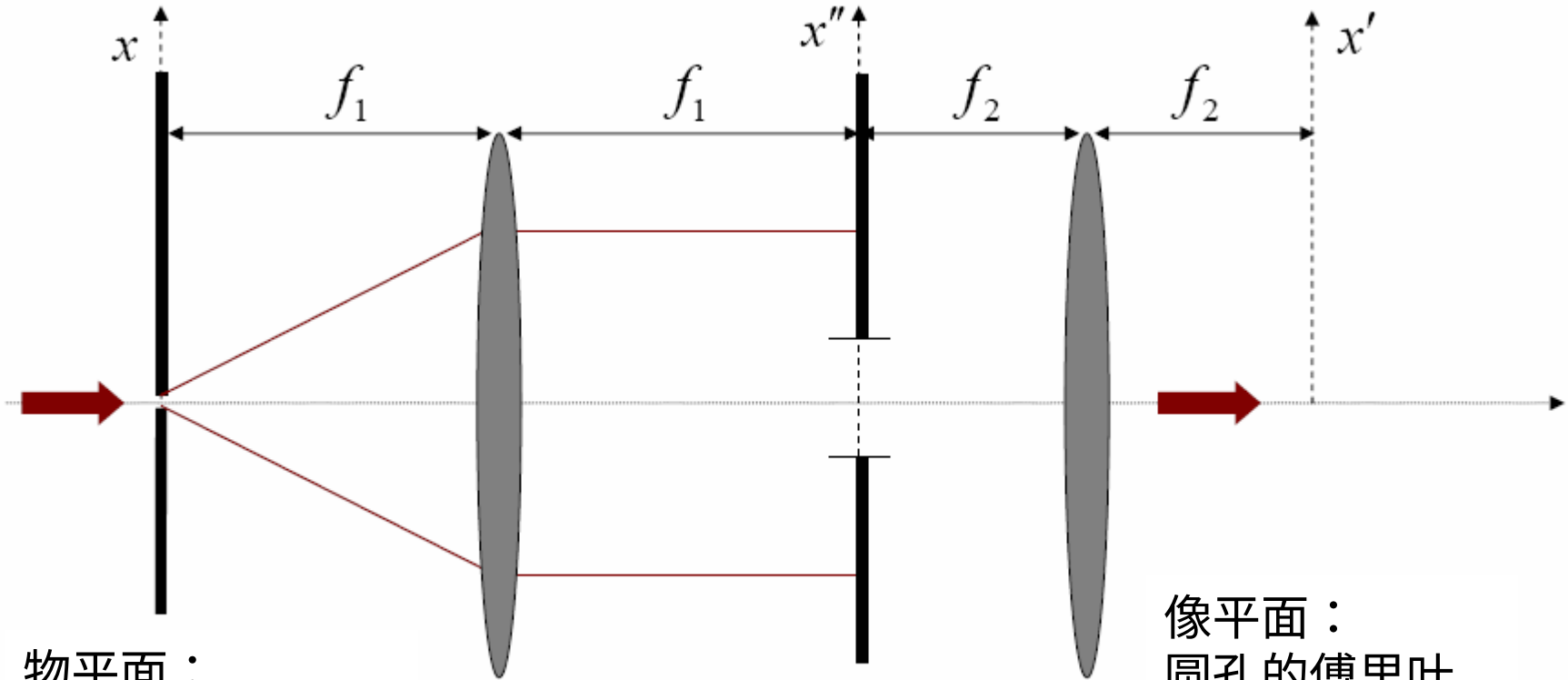
傅里叶滤波之后的强度



输出面强度



脉冲响应的形成



物平面：
针孔产生球面波

傅里叶平面：
圆孔受限

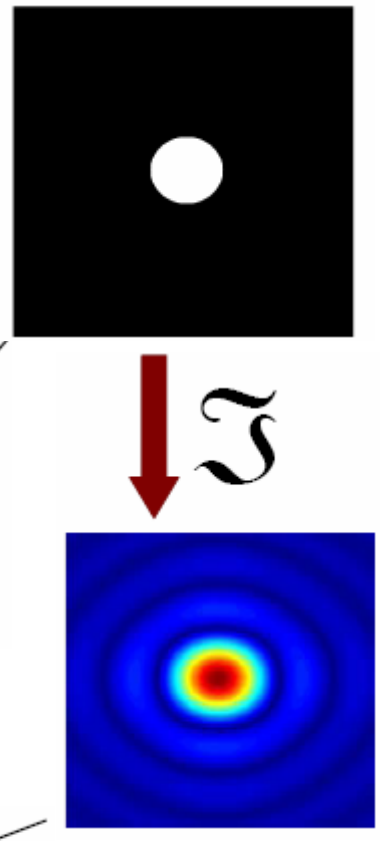
像平面：
圆孔的傅里叶
变换，爱里斑

（平面波被剪切）

低通滤波

输入透明片后的场

$$g_{\text{in}}(x,y)=\delta(x)\delta(y)\Rightarrow G_{\text{in}}(u,v)=1$$



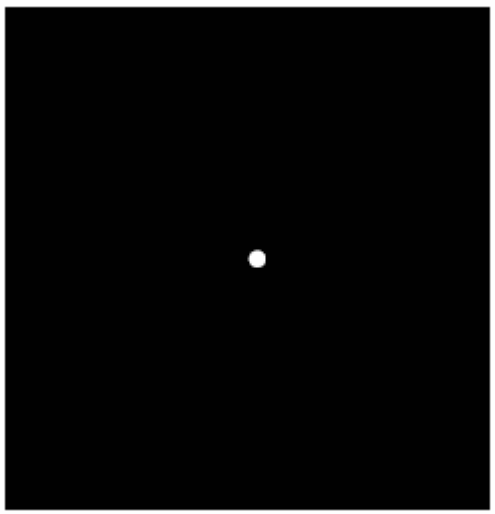
滤波前的场 $g_{\text{f}-}(x'',y'')=1$

滤波后的场 $g_{\text{f}+}(x'',y'')=\text{circ}\left(\frac{\sqrt{x''^2+y''^2}}{R}\right)$

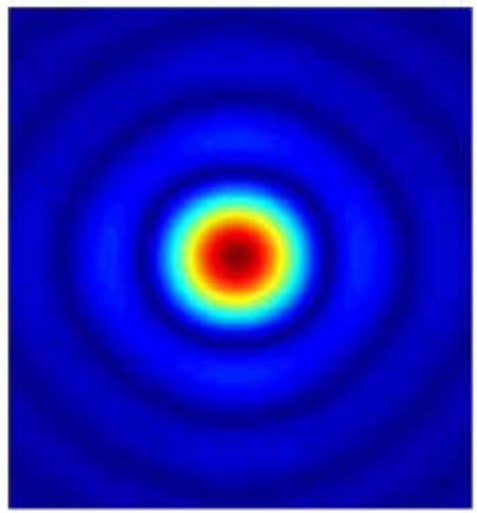
输出场（像平面） $G_{\text{out}}(u,v)=\text{circ}\left(\frac{\lambda f\sqrt{u^2+v^2}}{R}\right)\Rightarrow$

$g_{\text{out}}(x',y')\propto \text{jinc}\left(\frac{R\sqrt{x'^2+y'^2}}{\lambda f}\right)$ (爱里斑)

空间滤波效果



圆孔的傅里叶
平面滤波



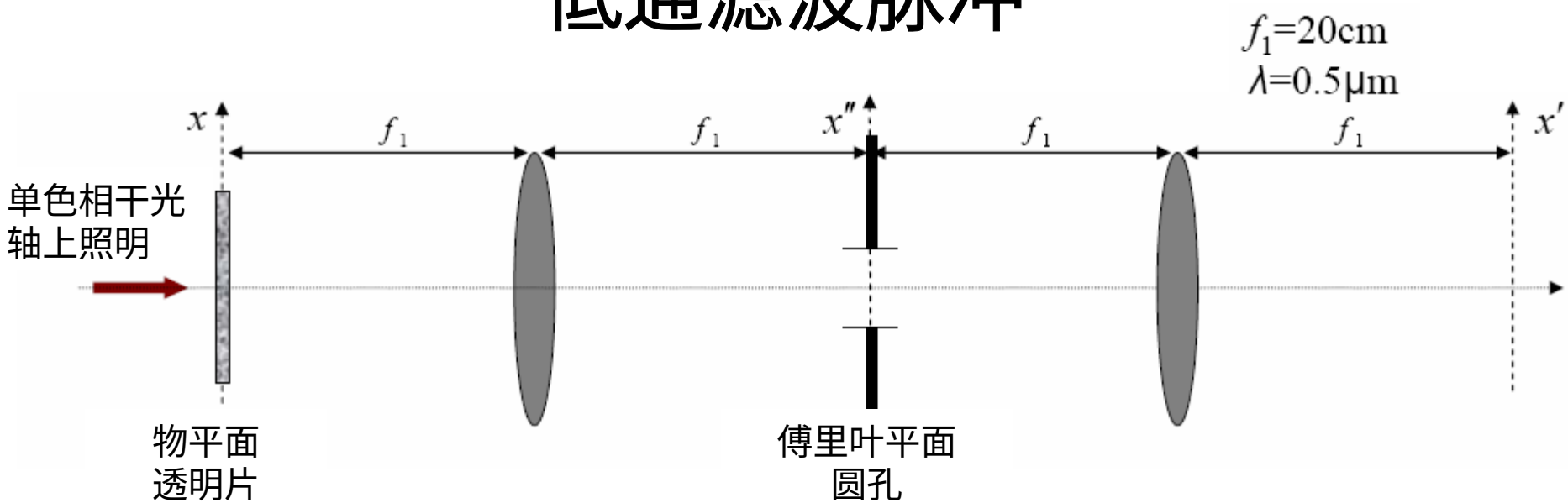
原物

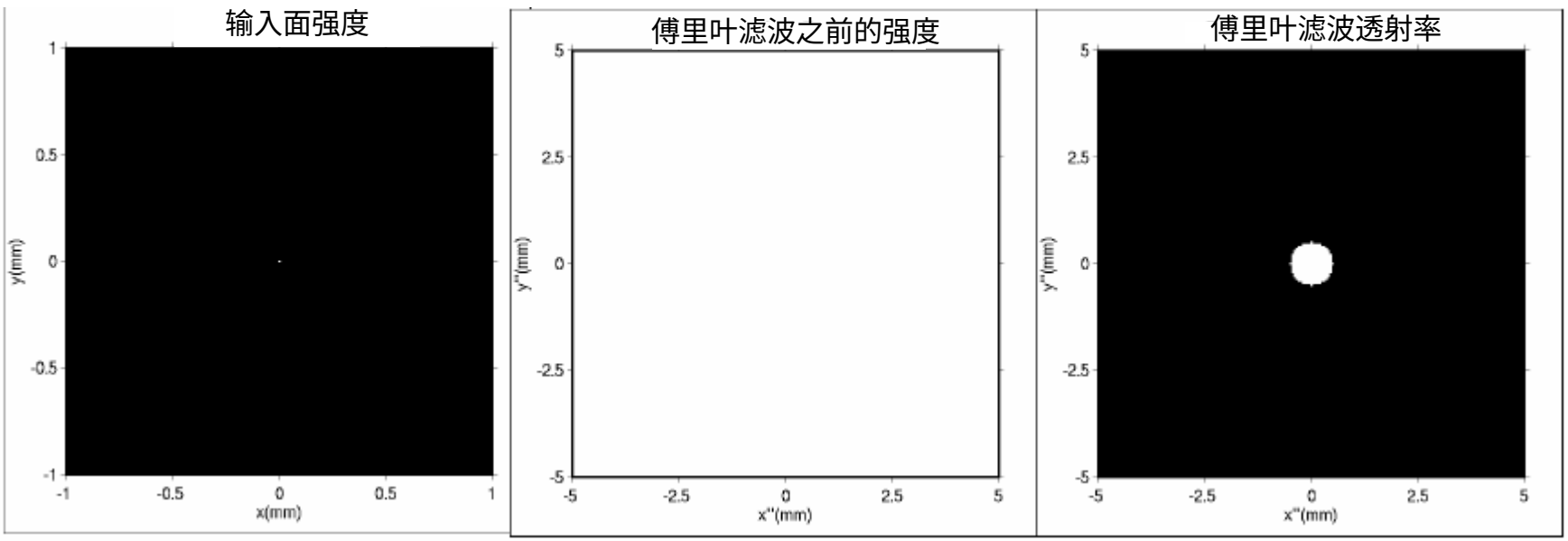
（小孔 ⇔ 脉冲，产生球面波通过透明片）

脉冲响应

（即点扩散函数，原始点物被模糊化成一个爱里斑，或jinc）

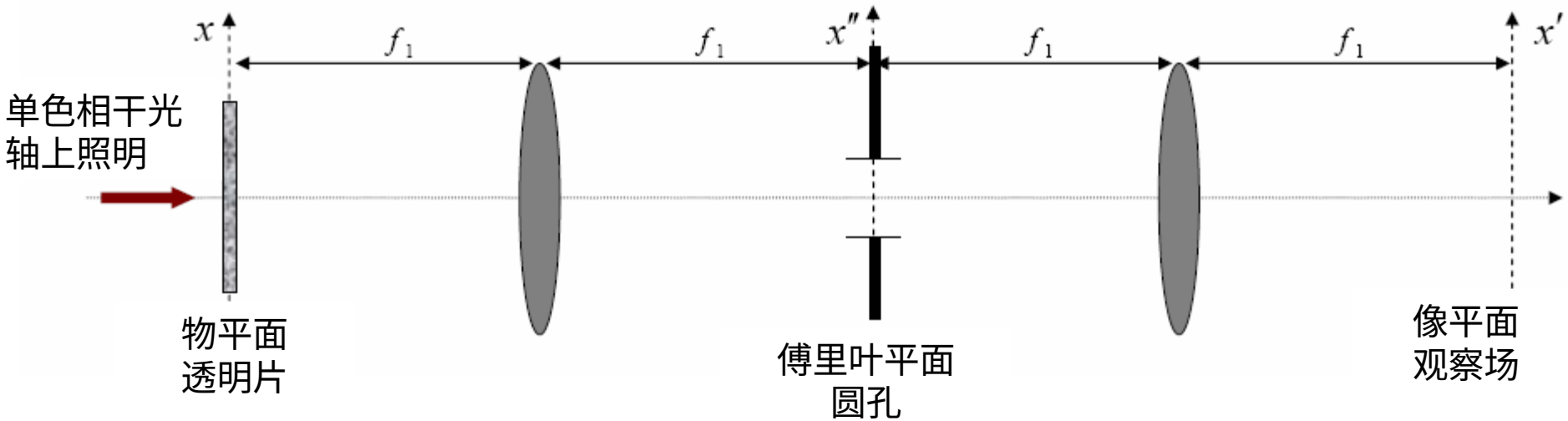
低通滤波脉冲


f_1=20cm
λ=0.5μm
x
f_1
f_1
x″
f_1
f_1
x′
单色相干光
轴上照明
物平面
透明片
傅里叶平面
圆孔


输入面强度
y(mm)
x(mm)
傅里叶滤波之前的强度
y″(mm)
x″(mm)
傅里叶滤波透射率
y″(mm)
x″(mm)

空间频率剪切

f_1=20cm
λ=0.5μm



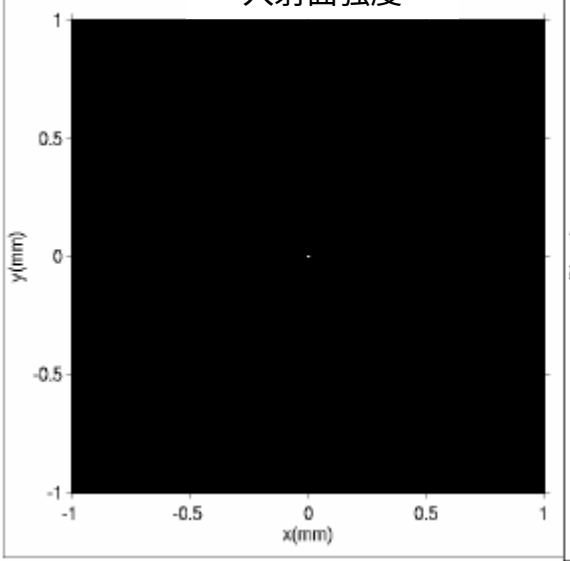
单色相干光
轴上照明

物平面
透明片

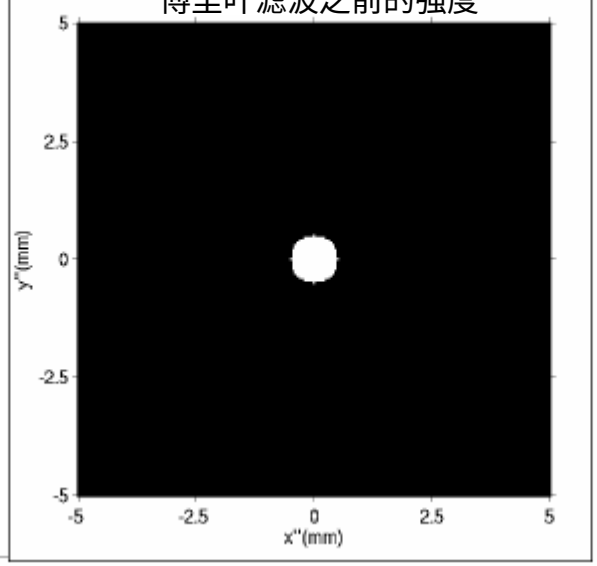
傅里叶平面
圆孔

像平面
观察场

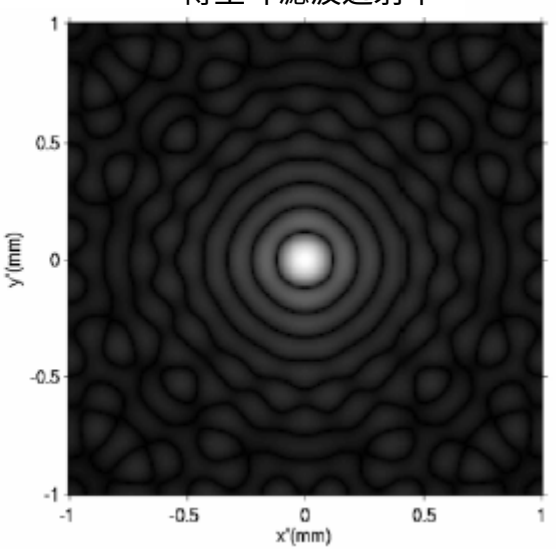
入射面强度



傅里叶滤波之前的强度

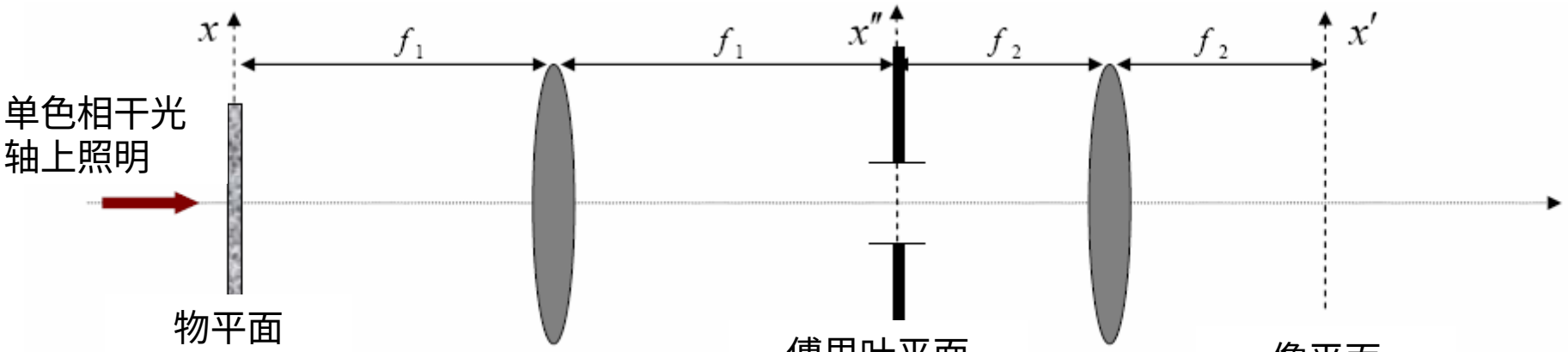


傅里叶滤波透射率



注：伪增强旁瓣

4F系统低通滤波



$g_{\text{in}}(x,y)$

$\Im$

傅里叶
变换

到达傅里
叶平面上
的场

$$H(x'',y'') = \text{circ}\left(\frac{\sqrt{x''^2+y''^2}}{R}\right)$$

$$G_{\text{in}}\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)$$

离开傅里叶
平面的场

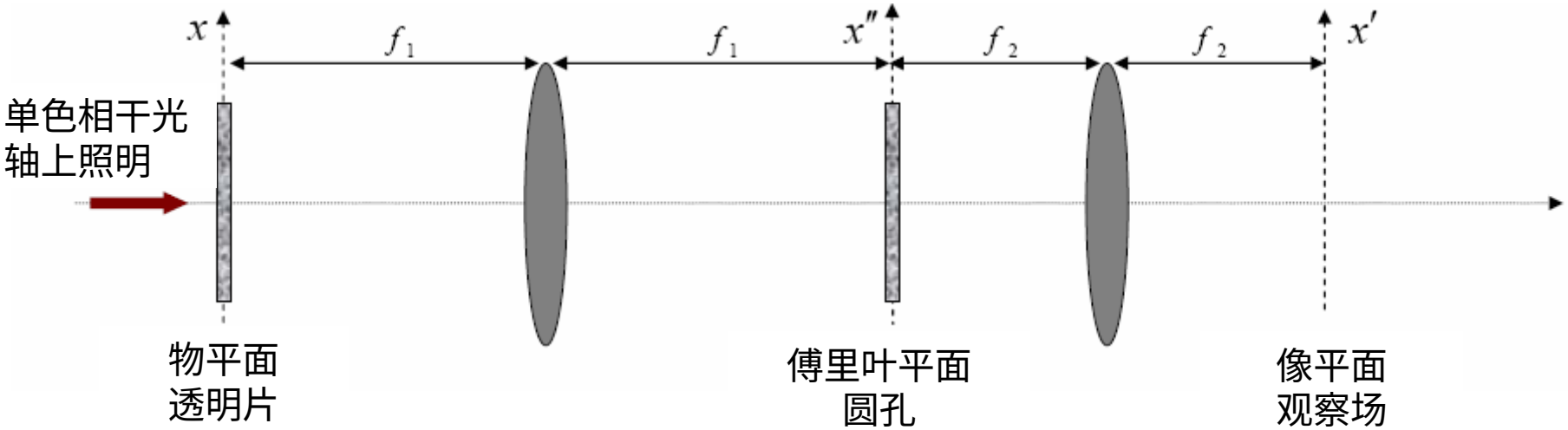
$$G_{\text{in}}\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)\times\text{circ}\left(\frac{\sqrt{x''^2+y''^2}}{R}\right)$$

$\Im$

傅里叶
变换

$$(g_{\text{in}} * \text{jinc})\left(-\frac{f_1}{f_2}x',-\frac{f_1}{f_2}y'\right)$$

4F系统的空间滤波



$g_{\text{in}}(x,y)$

$\Im$

到达傅里叶平面上的场

傅里叶变换

$H(x'',y'')$

$G_{\text{in}}\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)$

离开傅里叶平面的场

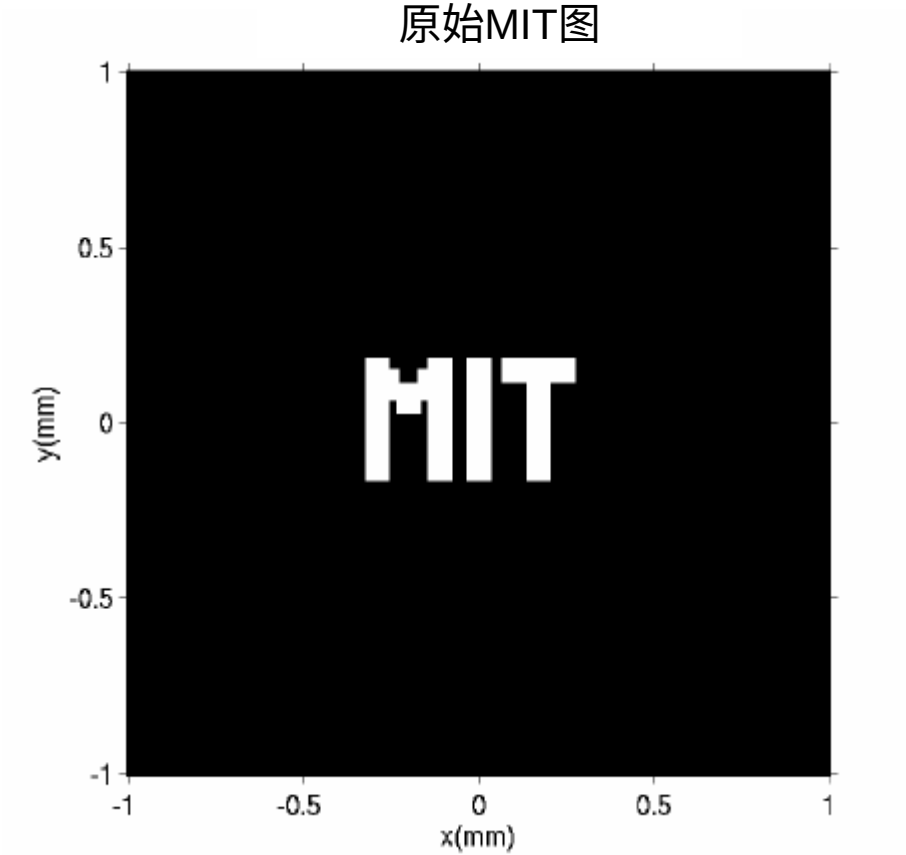
$G_{\text{in}}\left(\frac{x''}{\lambda f_1},\frac{y''}{\lambda f_1}\right)\times H(x'',y'')$

$\Im$

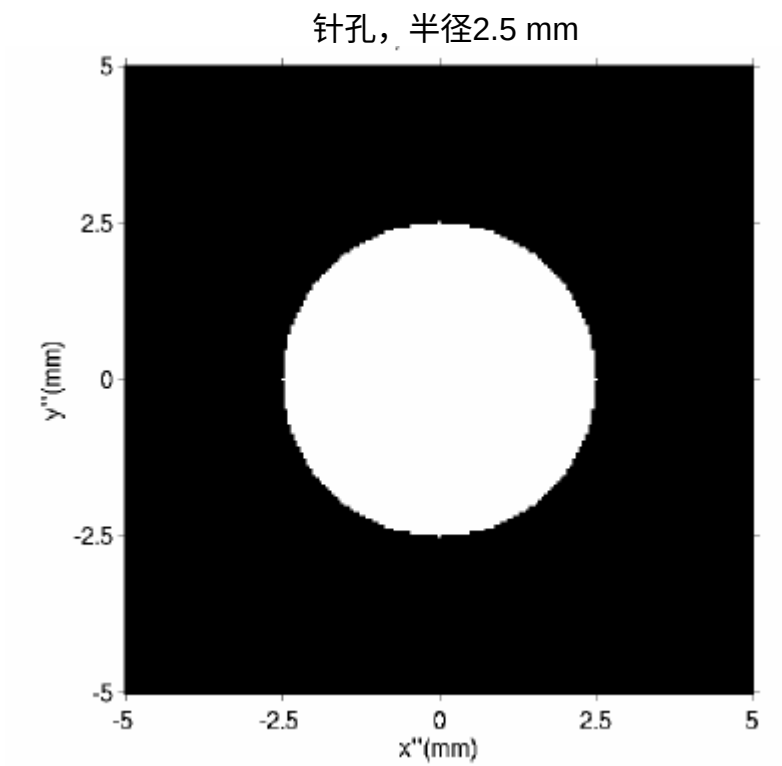
傅里叶变换

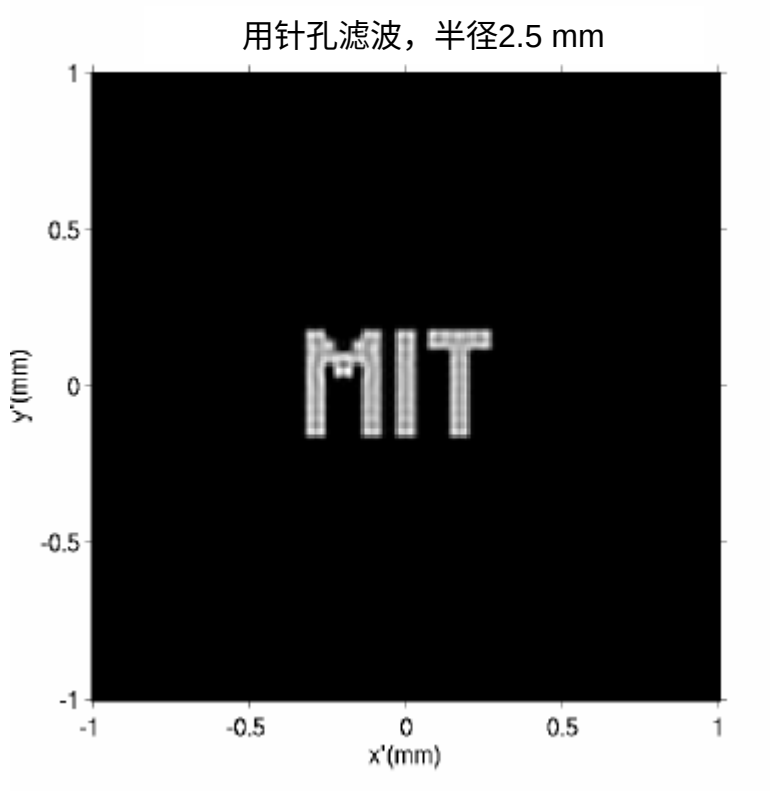
$(g_{\text{in}} * h)\left(-\frac{f_1}{f_2}x',-\frac{f_1}{f_2}y'\right)$

范例：振幅型MIT图样



弱低通滤波





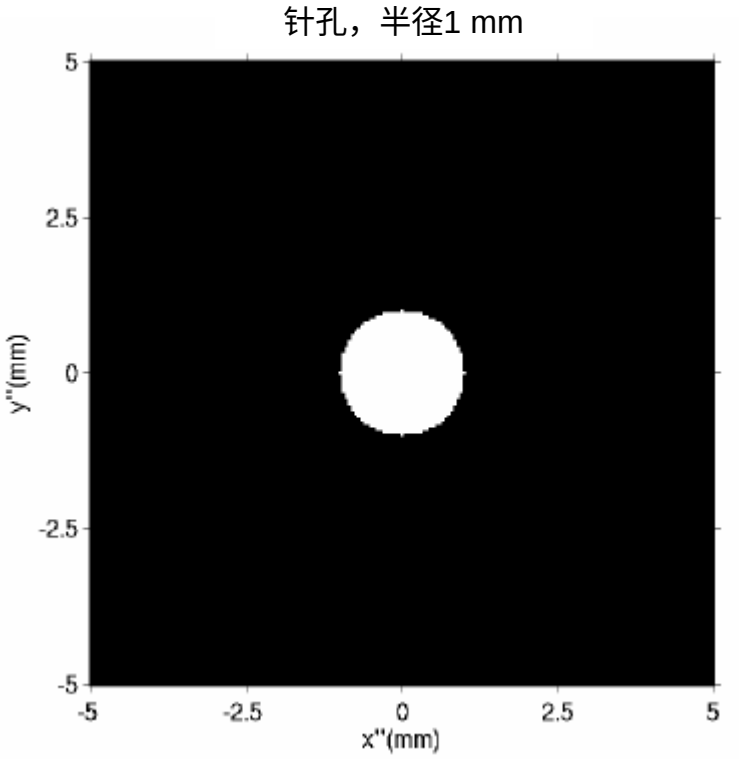
f_1=20cm
λ=0.5μm

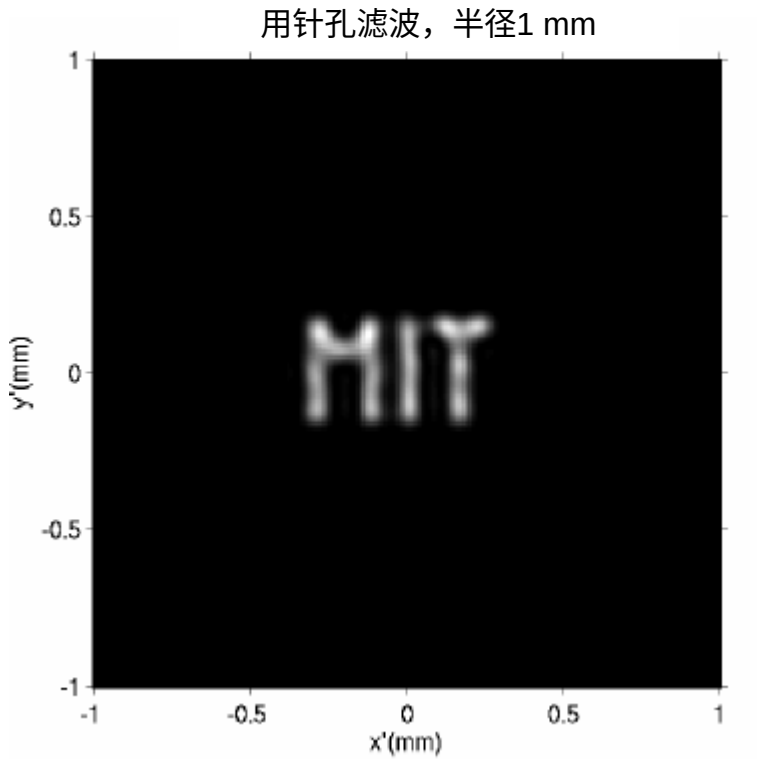
傅里叶滤波器

像面上的强度

中度低通滤波

（即模糊化）



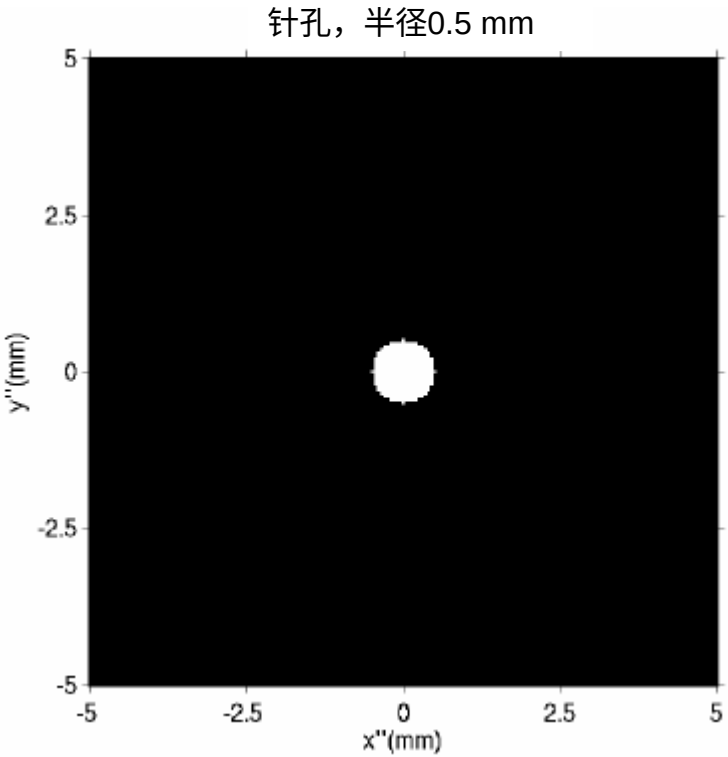


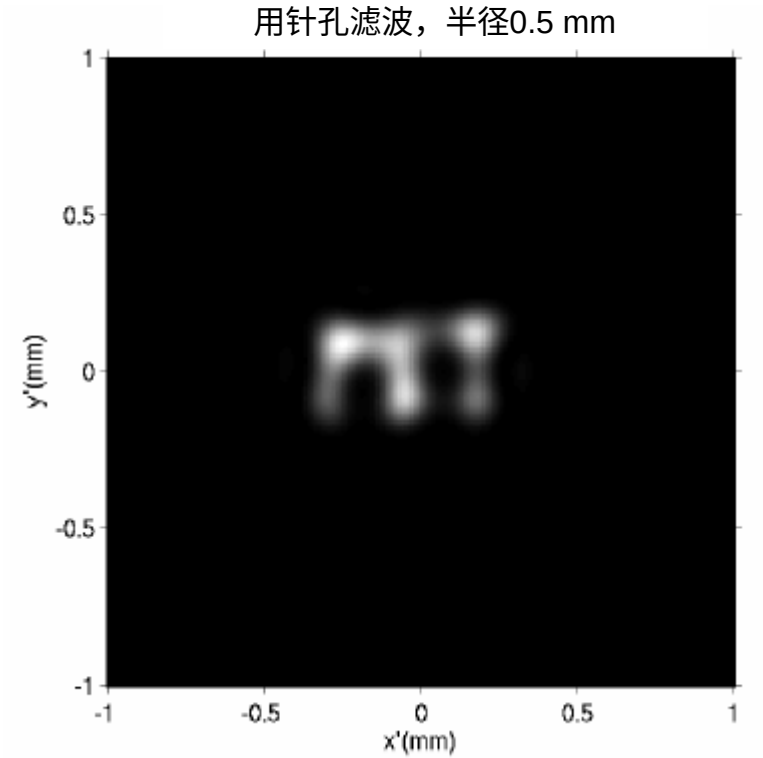
f_1=20cm
λ=0.5μm

傅里叶滤波器

像面上的强度

强低通滤波



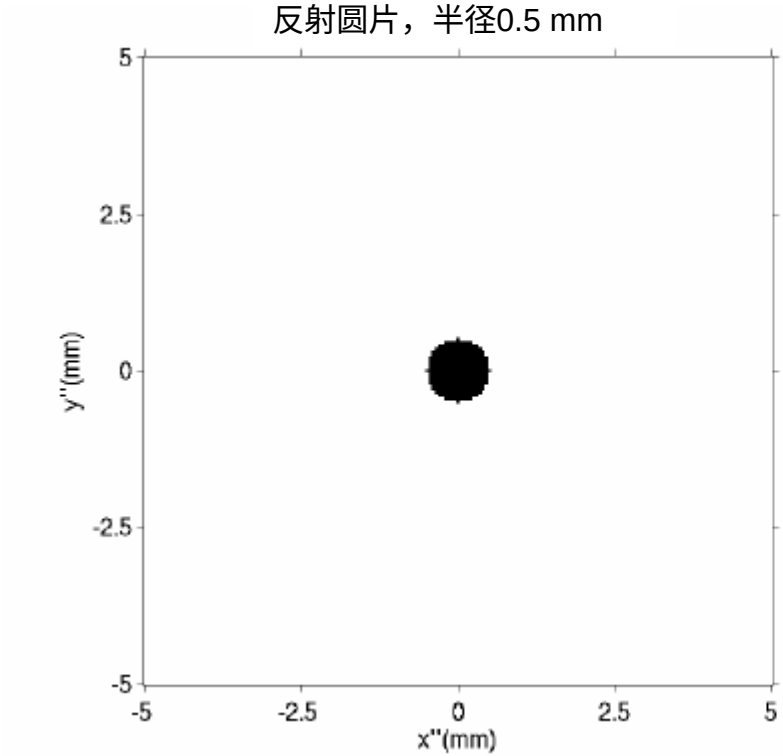


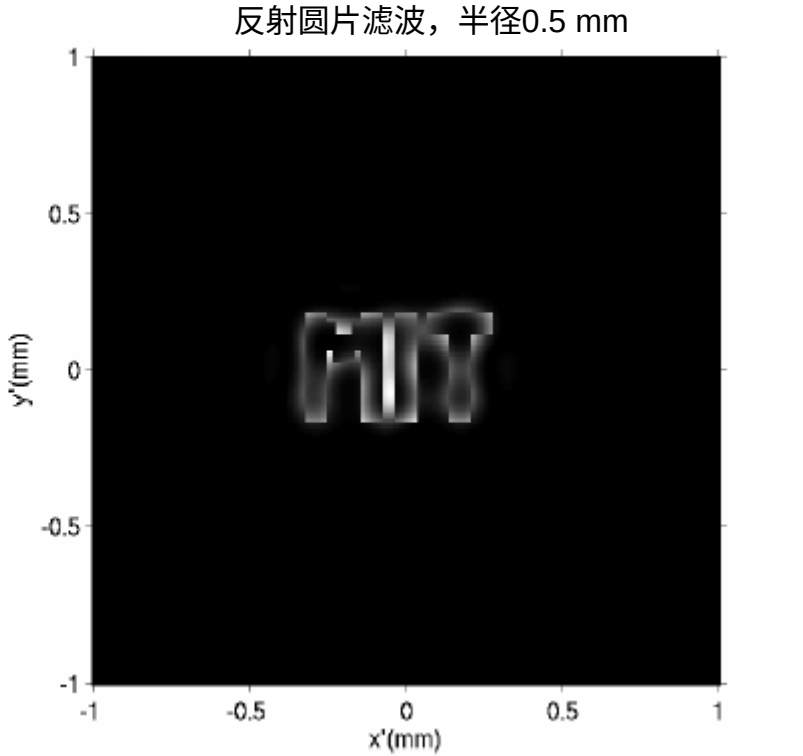
f_1=20cm
λ=0.5μm

傅里叶滤波器

像面上的强度

中度高通滤波





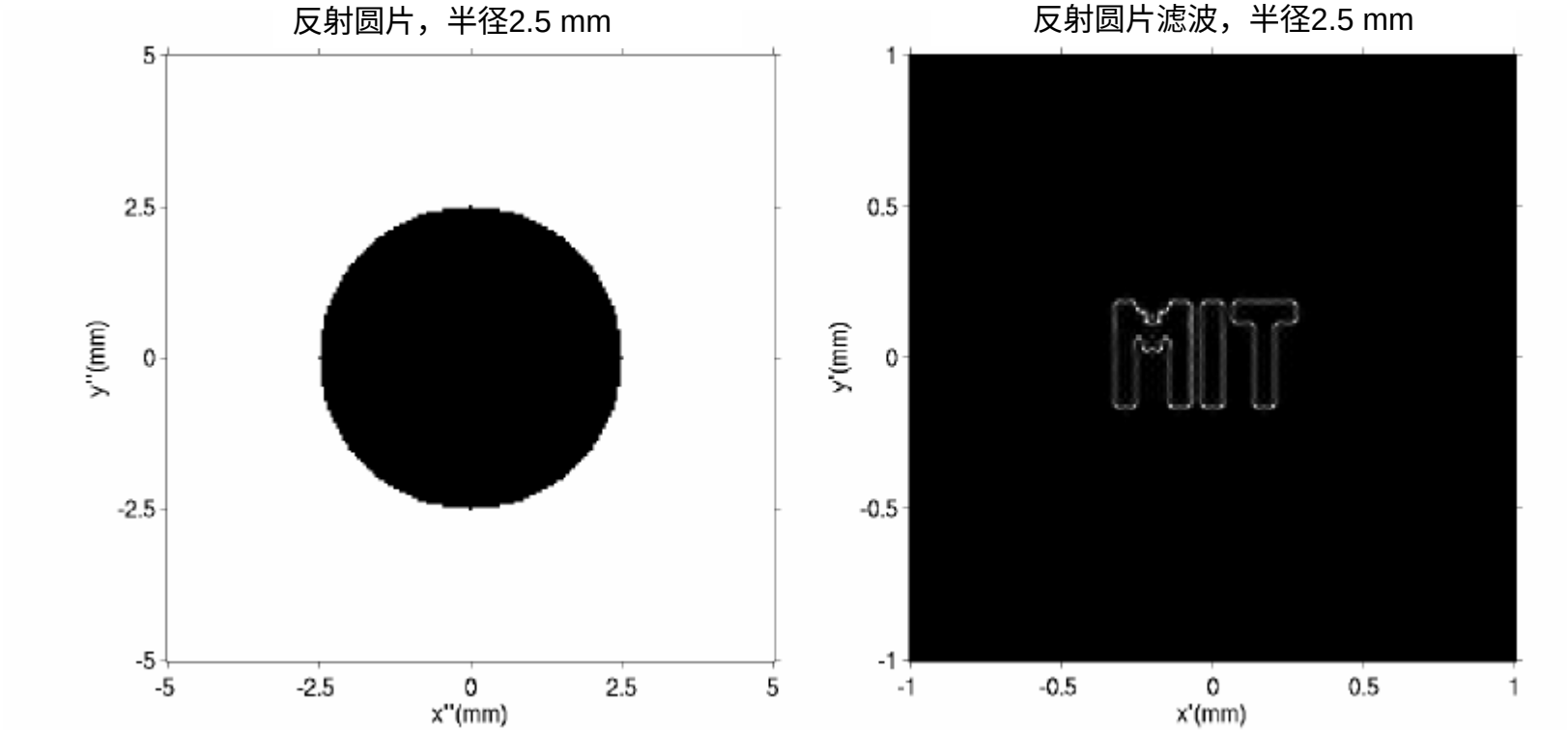
f_1=20cm
λ=0.5μm

傅里叶滤波器

像面上的强度

强高通滤波

（边缘增强）

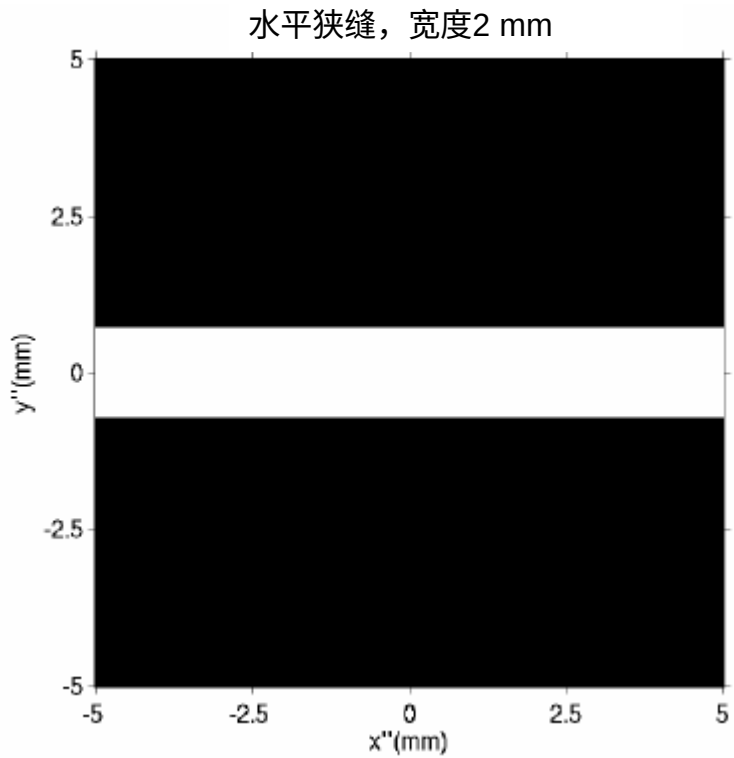


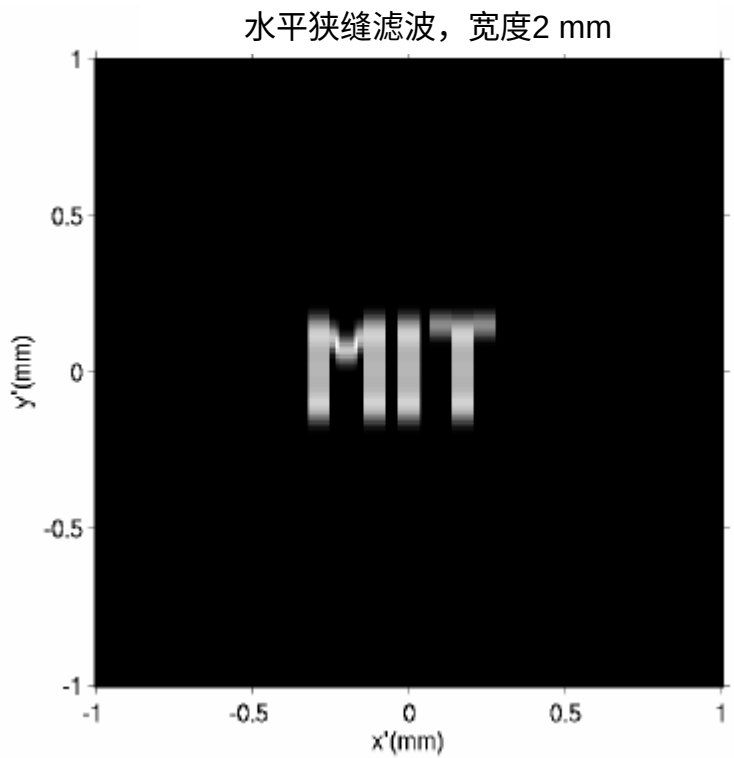
f_1=20cm
λ=0.5μm

傅里叶滤波器

像面上的强度

一维模糊



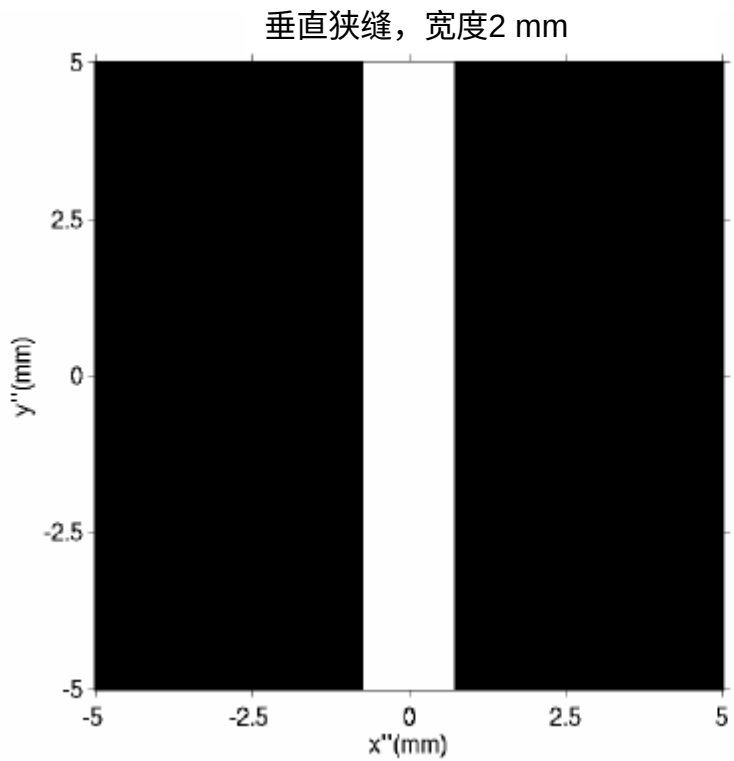


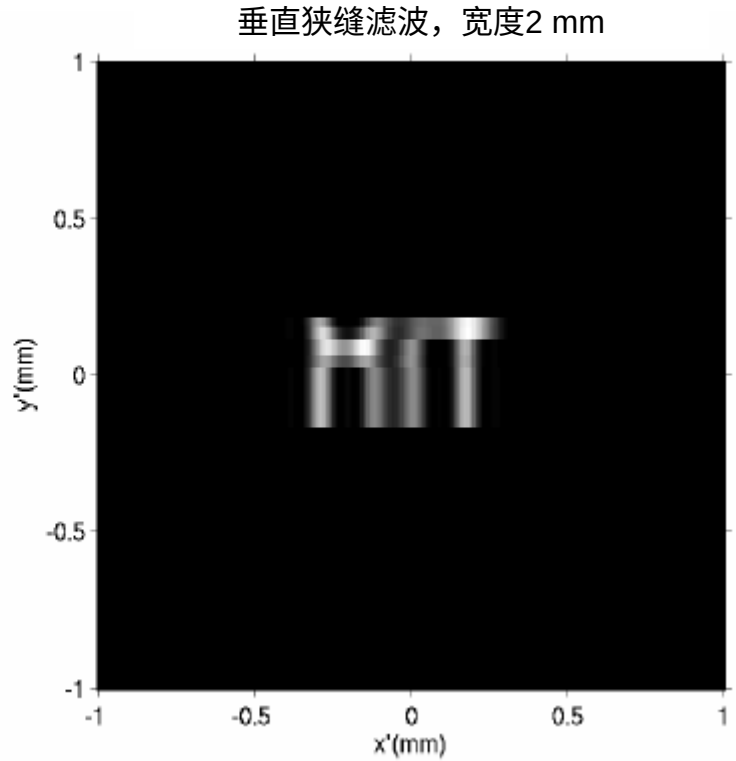
f_1=20cm
λ=0.5μm

傅里叶滤波器

像面上的强度

一维模糊



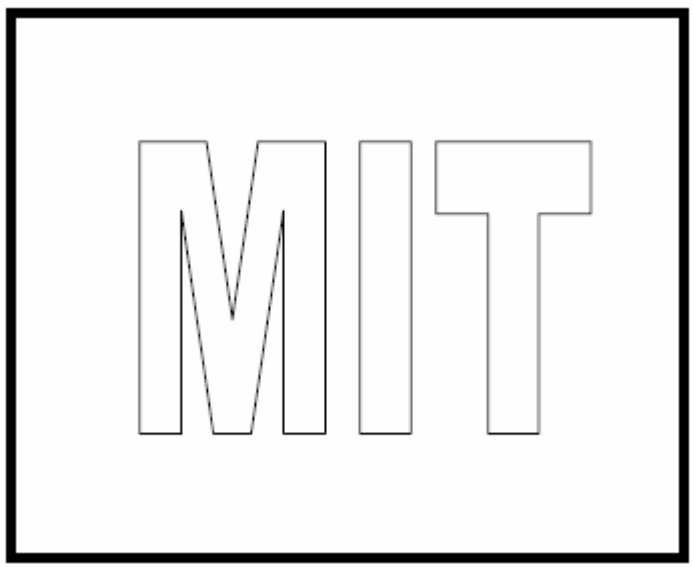


f_1=20cm
λ=0.5μm

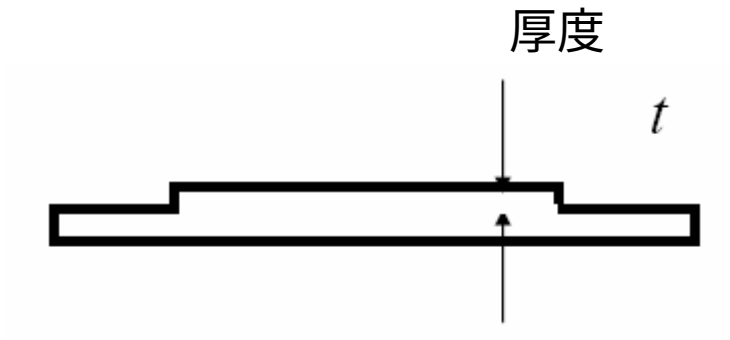
傅里叶滤波器

像面上的强度

位相物体



玻璃片
（透明）

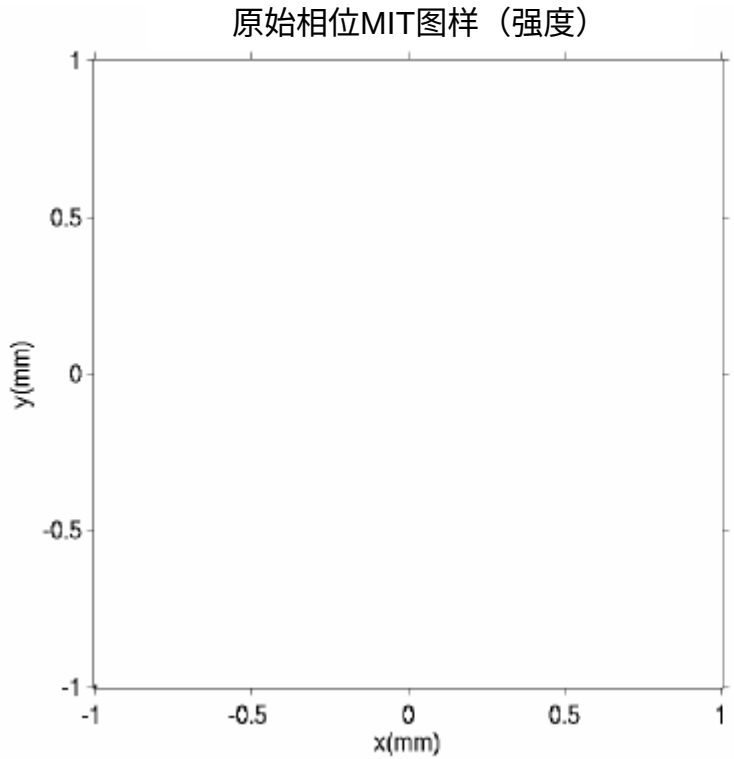


突出部分相移量 $\varphi = 2\pi(n-1)t/\lambda$

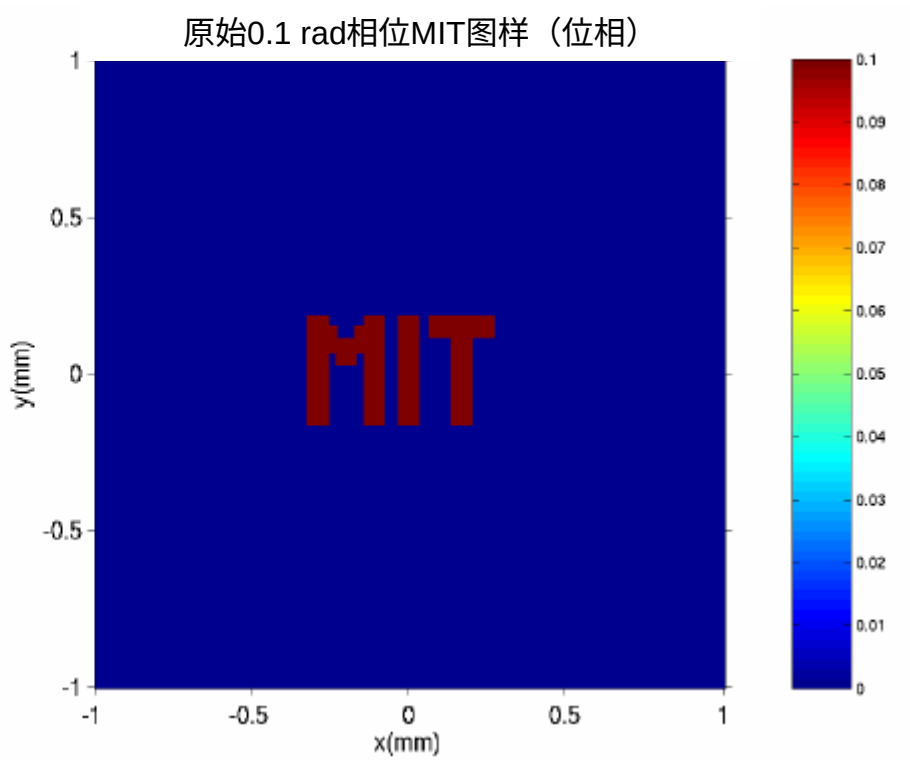
相干照明

通常用于生物物体成像（细胞等）

观察位相物体

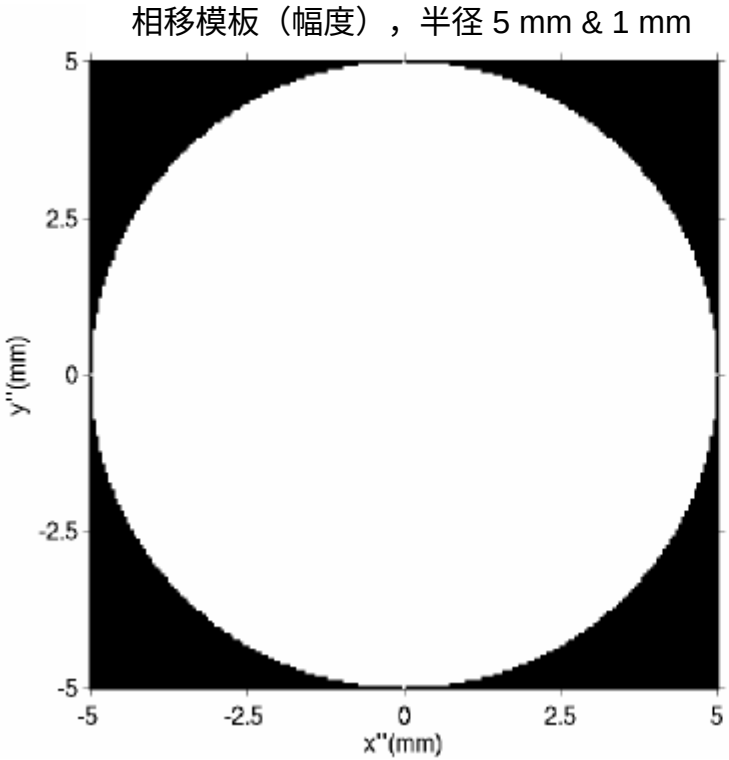


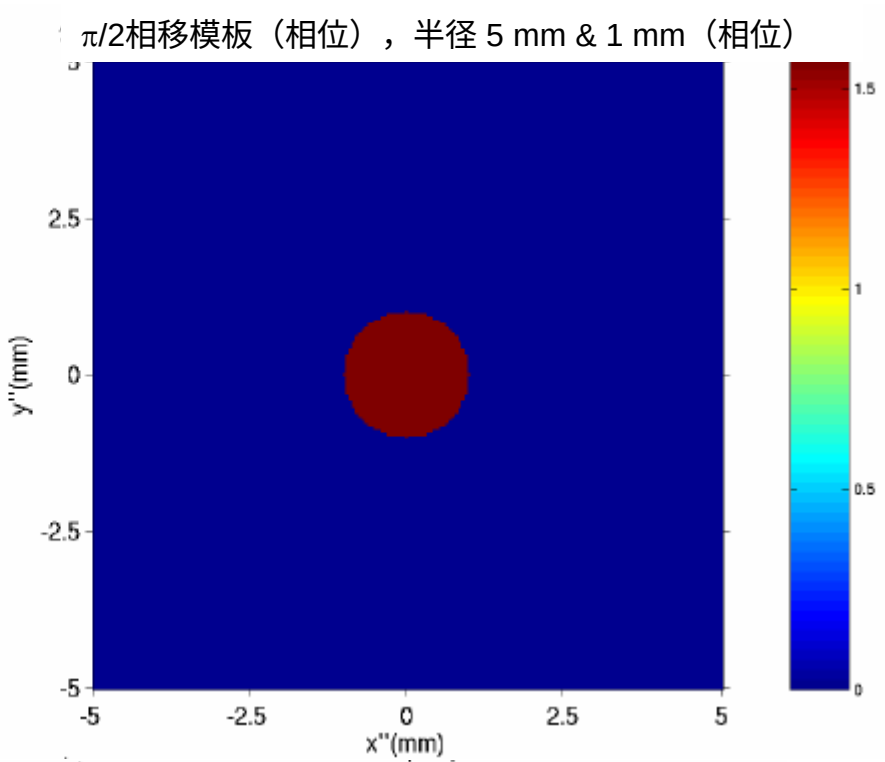
强度
（物体不可见）

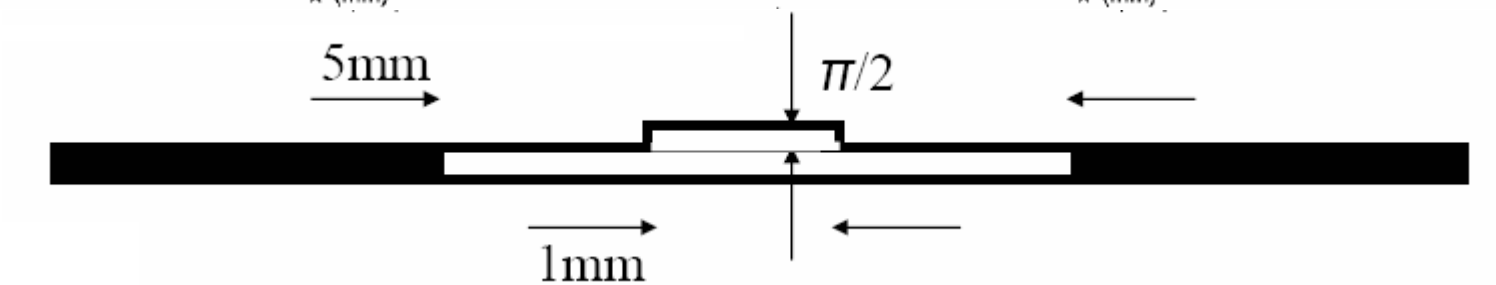


振幅
（需要干涉仪）

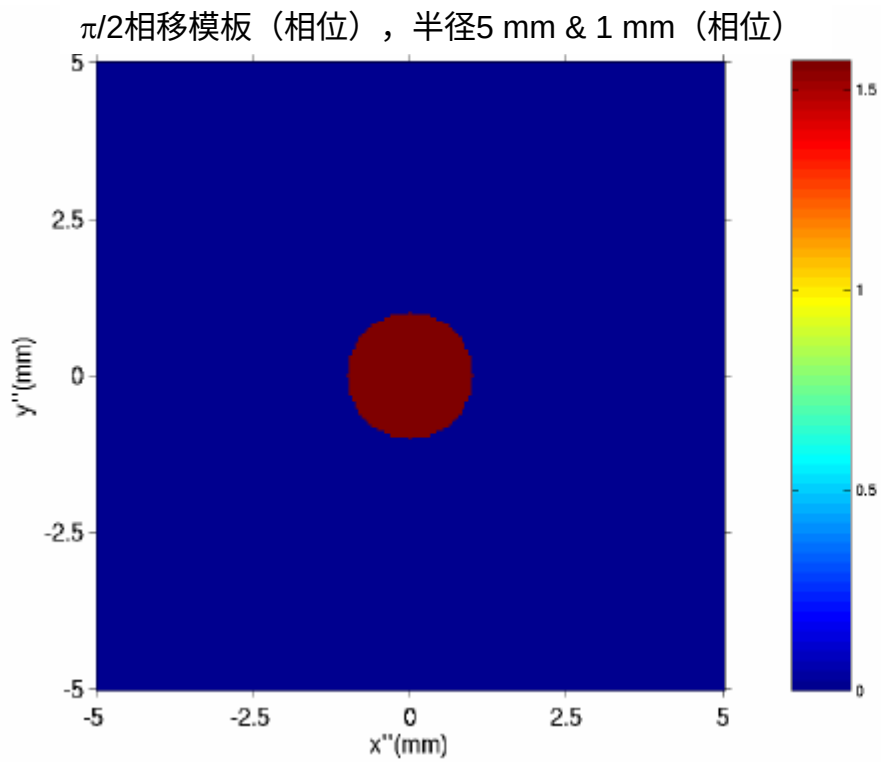
泽尼克相移模板

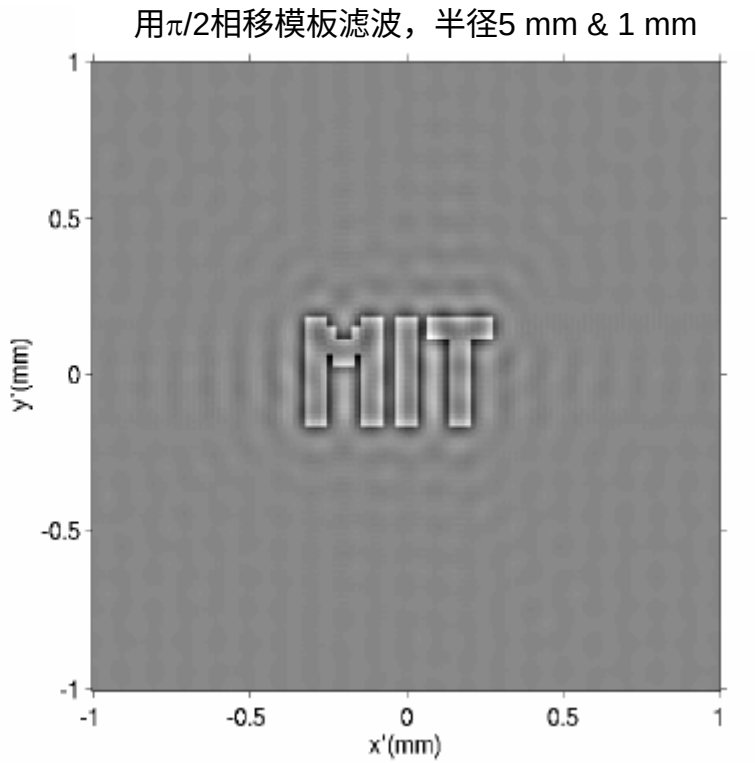

相移模板（幅度），半径 5 mm & 1 mm
5
2.5
0
-2.5
-5
y''(mm)
-5
-2.5
0
2.5
5
x''(mm)


π/2相移模板（相位），半径 5 mm & 1 mm（相位）
2.5
0
-2.5
-5
y''(mm)
-5
-2.5
0
2.5
5
x''(mm)
1.5
1
0.5
0


5mm
π/2
1mm

用泽尼克模板成像



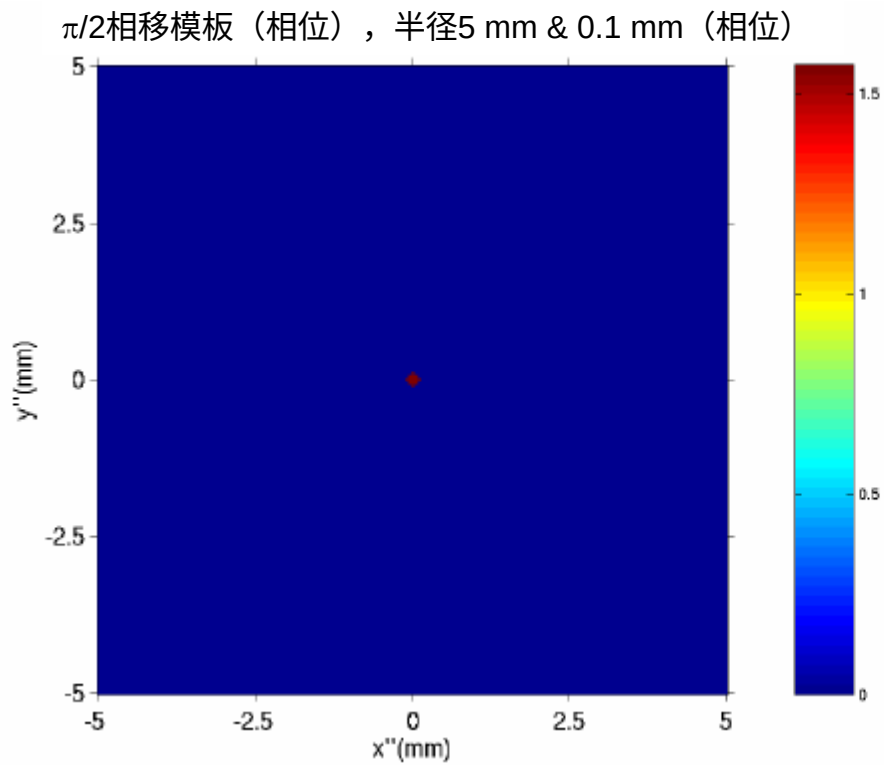


f_1=20cm
λ=0.5μm

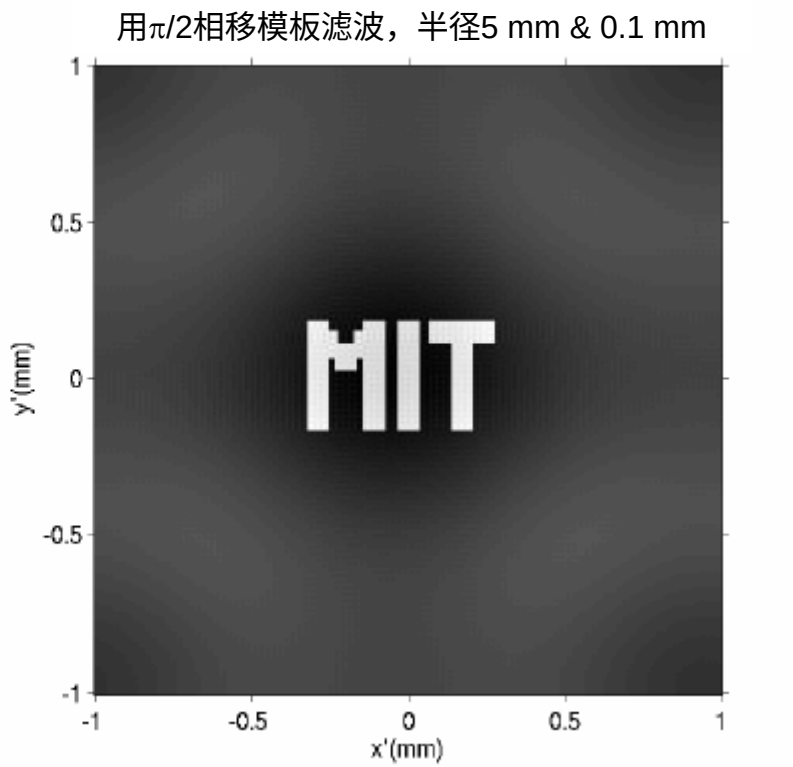
傅里叶滤波器

像面上的强度

用泽尼克模板成像



傅里叶滤波器



像面上的强度

f_1=20cm
λ=0.5μm